

# Economic Nonlinear Model Predictive Control for Mechanical Pulping Processes

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**Abstract**—In this paper we present an economic model predictive control (*econ* MPC) strategy for a two-stage (primary and secondary refining) Mechanical Pulping (MP) process. The MP process is a complex multi-input multi-output (MIMO) nonlinear process with strong interactions among the variables. In order to guarantee both stability and convergence of the closed-loop nonlinear MP process, two different *econ* MPC schemes are proposed: one with penalty on the increment of the input and one with penalty on the offset of the input from its steady-state target. We demonstrate that both *econ* MPC schemes achieve significant amount of energy reduction in terms of the specific energy consumed by the process while ensuring closed-loop stability and convergence to a nearby steady-state. In addition, we show that more energy reduction can be achieved by using the *econ* MPC with penalty on the input increment compared with the other scheme. Simulation results also demonstrate the potential benefits of using *econ* MPC over the standard MPC technique.

## I. INTRODUCTION

The mechanical pulping (MP) process plays an important role in the pulp and paper industry. In recent years, this industry has faced a number of new challenges due to a.) the increasing electrical energy prices. About 1/3 the cost of mechanical pulp is electrical energy which is rapidly increasing in recent years. This increase is driving the industry into new process and controller designs that make it significantly low energy. b.) the changing of the demand markets and products. In recently years, the demands for newsprint and other printing and writing paper are rapidly declining at a rate of more than 10% a year, while the demand for packaging and absorbent products continues to increase globally. This results in MP mills moving into products that have dramatically different quality requirements than conventional products. With the aforementioned reasons, it is imperative for the MP mills to develop advanced process control techniques to reduce electrical energy consumption and adapt to the new market demands. In the MP process, high consistency wood chip refiners, which are extensively used to break wood chips into individual fibres, play a central role in producing high-quality pulp. Extensive research efforts have been made over the past two decades towards refiner optimization,

cost reduction and product quality improvement in the MP plant [1]–[4]. However, the advanced control of today’s mechanical pulping processes is still relatively basic [5].

In the recent decades, Model Predictive Control (MPC) has become one of the most successful techniques which have made a significant impact on control of the process industry [5]–[7]. A number of research results on controlling MP processes by using MPC strategy have been reported [8]–[10]. Recent advances within process economic optimization and process control have focused on nonlinear model predictive control, which has provided an extension to include the nonlinear models and non-convex cost function for trajectory tracking and dynamic optimization [11]–[13]. More recently, the authors in [14]–[16] presented some results on the control and optimization of MP processes using nonlinear MPC.

In order to ensure the stability of the closed-loop systems, in the standard linear MPC theory, the deviation of the predicted state from the desired state is penalized. However, it has been observed that the classical methods for the MPC analysis can not be extended to the *econ* MPC when nonlinearity is present. It is known that due to the potential nonconvexity of the costs as well as the nonlinearity of the underlying dynamic system, convergence is not always achieved. Some work has been done in the past few years to analyze the convergence and stability of the nonlinear MPC system [17]–[20].

In this paper, we attempt to develop an economic model predictive control algorithm for MP processes to reduce the energy cost and guarantee the closed-loop stability and convergence. It is assumed that all the variables in the MP processes are measurable. The MP process considered in this work has two-stages of refining with a primary and a secondary refiner. Specifically, in the proposed approach, the economic MPC minimizes the total specific energy, which is related directly to desired economic considerations of the pulp mill and is not necessarily dependent on the steady-state. However, to enforce convergence, two strategies with modified objective functions in *econ* MPC schemes are proposed and compared in the simulation part.

## II. MP PROCESS DESCRIPTION

### A. MP process

In general, a typical MP process consists of three consecutive operation stages: wood-chip pretreatment, chip refining and pulp processing. Among these three stages, the chip refining section plays a vital role in producing high-quality pulp and takes up to 60% of the total electricity consumption in pulp manufacturing [16]. Therefore, it is important to

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develop a control algorithm to improve the pulp quality and reduce the energy consumption associated with this stage.

Specifically, the process of wood chip refining is completed through two connected refiners: a primary and a secondary refiner. After the pretreatment, the wood chips are fed to the primary refiner, broken down to smaller particles, and finally into individual fibres as they move through the refining zone. In the secondary refiner, the fibres will be further refined according to the pulp quality requirements. It should be pointed out that both the primary and secondary refiner are working at high consistency (HC), where the consistency is defined as the mass ratio of the dry fibre to the mixture of dry fibre and water. High consistency refers to the ratio between 20% and 50% whereas the low consistency (LC) refers to the ratio between 3% and 5%.

In the past decade, a number of efforts have been made to develop control techniques to reduce the energy consumption and improve the pulp quality in the chip refining unit. However, most of these trials failed due to the lack of consideration of the high-nonlinearity inherent in this process [8], [16].

### B. Two-stage HC refining model

So far, various dynamic models for the MP process have been proposed and investigated in the literature [8], [14]–[16]. The model derived in this paper is based on the model in [15], [16]. The model of the two-stage MP refiners is used for both simulation and prediction in the *econ* MPC. The disturbances in the chips, such as wood species, density, the size distribution and moisture are also considered in the simulation. To be specific, the following key variables and models are used to develop the discrete-time nonlinear model for the two-stage MP process.

1) *Production rate*: Production rate or throughput of a MP process is defined as the wood flow fed to the refiner [8]. We use the following model for the production rate  $P$  (tonnes/day):

$$P = k_a \cdot k_p \cdot S_c \cdot d_c \cdot R, \quad (1)$$

where  $k_a$  is a constant parameter which is obtained from the mill data of each particular production line.  $k_p$  ( $m^3/rev$ ) is the proportional constant.  $S_c$  (%) is the chip solid content.  $d_c$  ( $kg/m^3$ ) is the chip bulk density.  $R$  ( $rpm$ ) is the transfer screw speed.

2) *Motor load*: The motor load or net power is defined as the electrical energy delivered to the fibre flow in the refiner. Motor load may vary with changes in throughput, plate gap, refining consistency or raw material quality. As in [16], the motor load model employed in this work is described as follows,

$$M_i = \frac{km_i \cdot P}{D_i} (1 - e^{(-10G_i)})(a_i - b_i \cdot G_i), \quad (2)$$

where  $M_i$  (MW),  $i = 1, 2$ , is the motor load for the  $i$ -th refiner.  $D_i$  ( $l/min$ ) is the dilution water flow rate.  $G_i$  ( $mm$ ) is the gap size. The plate gap accuracy is normally in the range of [0.05mm, 0.2mm].  $a_i$ ,  $b_i$ , and  $km_i$  are parameters of the

refiner, and they may vary according to the refiner's plate size and mechanical design.

3) *Consistency*: Given a specific energy, the consistency of the refining zone is known to have a major effect on the pulp qualities. As in [8], the consistencies after the primary refiner and the secondary refiner (denoted as  $C_p$  and  $C_s$ , respectively) are determined as follows,

$$C_p = \frac{100P}{P + k_a \cdot D_p - ke_p \cdot M_p}, \quad (3)$$

$$C_s = \frac{100P}{P/(0.01C_p) + k_a \cdot D_s - ke_s \cdot M_s}, \quad (4)$$

where  $k_a$  is defined the same as in (1).  $ke_p$  and  $ke_s$  are the parameters for the two HC refiners, respectively.

4) *Pulp properties*: In this paper, the pulp properties such as the Canadian Standard Freeness (CSF,  $ml$ ), the long fibre content (LFC, %) and the shive content (SC, %) are used to assess the quality of the pulp after each refiner. The implicit nonlinear relationship between these pulp properties and the state variable for each refiner is expressed as follows,

$$g(CSF_{p,s}, LFC_{p,s}, SC_{p,s}, P, M_{p,s}, C_{p,s}) = 0, \quad (5)$$

where the subscripts  $p$  and  $s$  are used to denote the primary and the secondary refiner, respectively.

5) *Specific energy*: Specific energy (MW/tonnes/day) is defined to be the total motor load over the production rate, where the total motor load is a sum of the primary HC refiner motor load and the secondary motor load,

$$\begin{aligned} \text{Specific Energy (SE)} &:= \frac{\text{Total motor load}}{\text{Production rate}}, \quad (6) \\ \text{Total motor load} &:= \text{Primary motor load} \\ &\quad + \text{Secondary motor load.} \quad (7) \end{aligned}$$

The specific energy has been well recognized as a reliable indicator of the pulp quality [4]. In the *econ* MPC design literature, especially those with the purpose of energy reduction in a MP process, the minimization of the specific energy is used as the optimization objective [14]–[16]. In this paper, the specific energy is  $(x(2) + x(4))/x(1)$ , where  $x(1)$ ,  $x(2)$ , and  $x(4)$  are the primary motor load, the secondary motor load and production rate, respectively.

Based on the variables defined above, now we are in a position to demonstrate the MP process model. The discrete-time nonlinear model for a two-stage MP process at time  $k$  can be written as,

$$x(k+1) = Ax(k) + h(x(k), u(k)) = f(x(k), u(k)), \quad (8)$$

$$0 = g(x(k), y(k)) \quad (9)$$

where  $x(k) \in \mathbb{R}^{n_x}$ ,  $y(k) \in \mathbb{R}^{n_y}$ , and  $u(k) \in \mathbb{R}^{n_u}$  represent the state variable, controlled output variable, and the manipulated variable, respectively.  $x(k)$ ,  $y(k)$  and  $u(k)$  are defined in (10) below.  $A \in \mathbb{R}^{n_x \times n_x}$  is the dynamic matrix.  $h(\cdot) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \mapsto \mathbb{R}^{n_x}$  is the nonlinear function which maps the input steady

state into the dynamic variable of the MP process.

$$\begin{aligned}
 x(k) &= \begin{bmatrix} \text{Production rate, } P(k) \\ \text{Primary motor load, } M_p(k) \\ \text{Primary consistency, } C_p(k) \\ \text{Secondary motor load, } M_s(k) \\ \text{Secondary consistency, } C_s(k) \end{bmatrix}, \\
 y(k) &= \begin{bmatrix} \text{CSF after primary refining, } CSF_p(k) \\ \text{LFC after primary refining, } LFC_p(k) \\ \text{SC after primary refining, } SC_p(k) \\ \text{CSF after secondary refining, } CSF_s(k) \\ \text{LFC after secondary refining, } LFC_s(k) \\ \text{SC after secondary refining, } SC_s(k) \end{bmatrix}, \\
 u(k) &= \begin{bmatrix} \text{Chip-transfer screw speed, } R(k) \\ \text{Primary refiner plate gap, } G_p(k) \\ \text{Primary dilution flow rate, } D_p(k) \\ \text{Secondary refiner plate gap, } G_s(k) \\ \text{Secondary dilution flow rate, } D_s(k) \end{bmatrix}. \quad (10)
 \end{aligned}$$

### III. ECONOMIC MPC OPERATION FOR MP PROCESS

A typical MPC block diagram for the MP process is illustrated in Fig. 1. The objective of the MPC design is to reduce the energy consumption, minimize the pulp quality variance and also guarantee the convergence and stability of the closed-loop MP process. In this section, the *econ* MPC strategy based on the nonlinear MP model will be developed. Two schemes enforcing the convergence to the equilibrium point are discussed and compared for the two-stage refining process.

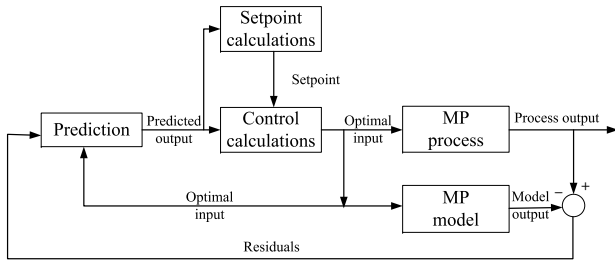


Fig. 1: Block diagram for model predictive control of a typical MP process

The *econ* MPC dynamic optimization problem is formulated for the nonlinear MP process described in equations (8) and (9) at sampling time  $k$  as follows. Here we assume that the current process state  $x(k)$  is available at the current time  $k$ .

$$V_N^0(x) := \min_{(X,U)} \sum_{i=1}^N \text{Specific energy}_i \quad (11)$$

$$\begin{aligned}
 \text{subject to} \quad & x(k|k) = x(k), \\
 & x(k+i|k) = f(x(k+i-1|k), u(k+i-1|k)), \\
 & 0 = g(x(k+i|k), y(k+i|k)), \\
 & x_{min} \leq x(k+i|k) \leq x_{max}, \\
 & y_{min} \leq y(k+i|k) \leq y_{max}, \\
 & u_{min} \leq u(k+i-1) \leq u_{max}, \quad i = 1, \dots, N, \quad (12)
 \end{aligned}$$

where the specific energy is defined in (6).  $x(k+i|k)$  and  $y(k+i|k)$  are the predicted state variables and the predicted controlled variables based on the current process information at time  $k$ .  $X = \{x(k|k), x(k+1|k), \dots, x(k+N|k)\}$  and  $U = \{u(k|k), u(k+1|k), \dots, u(k+N-1|k)\}$  are the predicted state variables and the manipulated variables.  $N$  is the prediction horizon, which is equal to the control horizon for simplicity.  $x_{min}$  and  $x_{max}$ ,  $y_{min}$  and  $y_{max}$ ,  $u_{min}$  and  $u_{max}$  are the lower bound and upper bound of the state variable, controlled output, and the manipulated variable, respectively.

#### A. Scheme A: Economic MPC in MP process with the penalty of increment input

For the conventional MPC design, the steady-state  $x_s$  and  $u_s$  are usually obtained from the static optimizer, in which a linear model is assumed. However, due to the nonlinearities in the real plant and the nonconvex cost functions of the MP dynamic optimization, the actual optimal operating regime for the given plant and constraints may fail to be the equilibrium from the upper layer. In this section, a modified economic cost function is proposed as,

$$\begin{aligned}
 V_N^A(x) &:= \min_{(X,\Delta U)} \sum_{i=1}^N \text{Specific energy}_i \\
 &+ \|x(k+i|k) - x_s(k+i)\|_{Q_x^A}^2 + \|\Delta u(k+i|k)\|_{Q_{\Delta u}^A}^2, \quad (13)
 \end{aligned}$$

where the  $\Delta u(k+i|k) = u(k+i|k) - u(k+i-1|k)$  is the increment of the manipulated variable.  $x_s$  is the steady-state which can be obtained by the static optimization or from the upper layer.  $X = \{x(k|k), x(k+1|k), \dots, x(k+N|k)\}$  and  $\Delta U = \{\Delta u(k|k), \Delta u(k+1|k), \dots, \Delta u(k+N-1|k)\}$  are the predicted state variables and the increment of manipulated variables.  $Q_x^A$ ,  $Q_{\Delta u}^A$  are positive semidefinite weighting matrices.

In this scheme, the increment of the input signal  $\Delta u(k)$  is considered in the objective function (13). The benefit of this term is to simplify the implementation procedure, as less steady-state plant information is required. Moreover, instead of enforcing the manipulated variables to track the steady-state target, more freedom will be introduced to the manipulated variables to minimize the specific energy by dynamic optimization, which will give rise to less energy consumption. In addition, in the MP process, the precision of the plate gap is normally between  $0.02mm$  and  $0.5mm$ . Thus, for the sake of safety and equipment physical limitations, the actuator movement should be penalized during the optimization.

**Remark 3.1** In order to guarantee the stability and the convergence of the closed-loop MP system, in scheme A, two convex terms are added to the original *econ* MPC cost function (11). We should mention that, in [18], the authors only add the input increment term to guarantee the convergence. However, in a MP process, the final pulp property target of the production line may vary according to the customers' requirements. As a consequence, the steady states  $x_s$  will change accordingly. Thus, the first convex term which penalizes the distance of the predict state  $x(k)$  from its steady state  $x_s(k)$  is also necessary in the objective function.

**B. Scheme B: Economic MPC in MP process with penalty on the distance from the steady state**

In order to stabilize the system, another commonly used method is to add penalization on the distance of manipulated and state variables from their steady-state targets as a convex term in the objective function (11) [18], [21]. In this method, the modified *econ* MPC objective function is as follows,

$$V_N^B(x) := \min_{(X,U)} \sum_{i=1}^N \text{Specific energy}_i + \|x(k+i|k) - x_s(k+i)\|_{Q_x^B}^2 + \|u(k+i|k) - u_s(k+i)\|_{Q_u^B}^2, \quad (14)$$

where  $x_s(k)$  and  $u_s(k)$  can be obtained by static optimization or from the upper layer.  $Q_x^B$ ,  $Q_u^B$  are the weighting matrices. The other variables and parameters are defined similarly as the ones in Scheme A.

**Remark 3.2.** Note that even though one of the most important features associated with *econ* MPC technique is that it can directly optimize the economic cost of operating the plant without requiring any information of the steady state. This might render the precomputed steady state unnecessary. However, the work reported on *econ* MPC mostly includes the steady state information into the analysis. For example, the *econ* MPC with a terminal constraint requires the state to be equal to the steady state at the end of the horizon [18]; The strong duality or dissipativity assumption requires that the system have *a priori* knowledge of the steady state in order to carry out the stability analysis [17].

#### IV. SIMULATION RESULTS

In this section, we present two simulations to demonstrate the applicability of the previous approach in the model predictive control of the nonlinear MP process. In Simulation I, we compare the control performance of adding two different penalties in the economic objective functions as proposed in Scheme A and B. In the second simulation, the energy savings using the economic MPC will be tested through a comparison with the conventional set point tracking MPC. In these simulations, the nonlinear MP model is modeled in AMPL (A Mathematical Programming Language), and the nonlinear optimization problem is solved using IPOPT (Interior Point OPTimizer).

**A. Simulation I: Comparison of *econ* MPC with different penalty methods**

For the two-stage MP process as (9), the dynamic optimization problem with the penalty terms in scheme A is formulated as follows,

$$\begin{aligned} V_N^A(x) &:= \min_{(X,\Delta U)} \alpha \sum_{i=1}^N \text{Specific energy}_i \\ &+ \beta \{ \|x(k+i|k) - x_s(k+i)\|_{Q_x^A}^2 + \|\Delta u(k+i|k)\|_{Q_{\Delta u}^A}^2 \} \\ &= \min_{(X,\Delta U)} \alpha \sum_{i=1}^N \{ (x_2(k+i|k) + x_4(k+i|k)) / x_1(k+i|k) \} \\ &+ \beta \{ \|x(k+i|k) - x_s(k+i)\|_{Q_x^A}^2 + \|\Delta u(k+i|k)\|_{Q_{\Delta u}^A}^2 \}, \end{aligned} \quad (15)$$

$$\begin{aligned} \text{subject to} \quad & x(k|k) = x(k), \\ & x(k+i|k) = f(x(k+i-1|k), u(k+i-1|k)), \\ & 0 = g(x(k+i|k), y(k+i|k)), \\ & x_{min} \leq x(k+i|k) \leq x_{max}, \\ & y_{min} \leq y(k+i|k) \leq y_{max}, \\ & u_{min} \leq u(k+i-1|k) \leq u_{max}, \\ & \Delta u_{min} \leq \Delta u(k+i-1) \leq \Delta u_{max}, \quad i = 1, \dots, N, \end{aligned} \quad (16)$$

where  $x(k)$ ,  $u(k)$ , and  $\Delta u(k)$  are the state variable, manipulated variable and the increment of the manipulated variable in the two-stage MP process.  $x_s(k)$  is the steady-state which can be obtained from static optimization.  $f(\cdot)$  and  $g(\cdot)$  are the nonlinear dynamic function of the process which are defined in (8) and (9).  $N = 10$  is the prediction horizon which is assumed to be equal to the control horizon.  $x_{min}$  and  $x_{max}$ ,  $y_{min}$  and  $y_{max}$ ,  $u_{min}$  and  $u_{max}$ ,  $\Delta u_{min}$  and  $\Delta u_{max}$  are the lower bounds and upper bounds of the states, pulp qualities, the manipulated variable, and the increments of the input, respectively.  $\alpha$  and  $\beta$  are the weighting constants of the specific energy relative to the penalty terms. In this simulation, we specify that  $\alpha = 1000$ ,  $\beta = 1$ . It should be mentioned here that, if the convergence speed is more important or in the industrial applications or one wants to force the states to reach the set-points, more penalty should be added to  $\beta$ . The weighting matrices  $Q_x^A$  and  $Q_{\Delta u}^A$  are chosen to be  $Q_x^A = \text{diag}([0.01, 10, 0.1, 10, 0.1]^T)$ ,  $Q_{\Delta u}^A = \text{diag}([0.1, 100, 0.01, 100, 0.01]^T)$ , respectively.

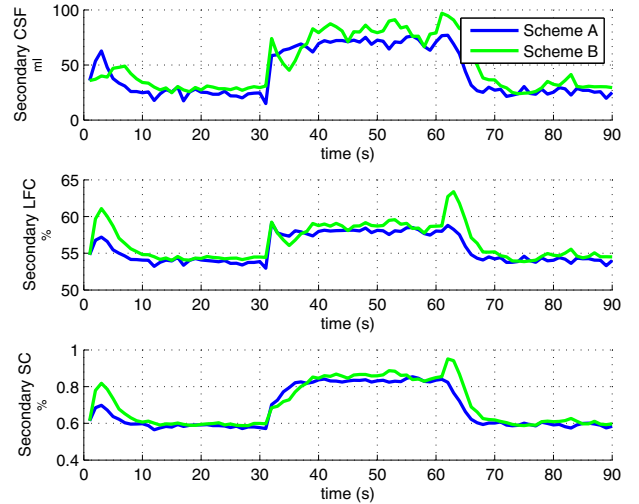


Fig. 2: Pulp quality after two-stage HC refining by using *econ* MPC in simulation I

As a comparison, the modified *econ* MPC optimization in scheme B is as follows,

$$V_N^B(x) := \min_{(X,U)} \alpha \sum_{i=1}^N \text{Specific energy}_i + \beta \{ \|x(k+i|k) - x_s(k+i)\|_{Q_x^B}^2 + \|u(k+i|k) - u_s(k+i)\|_{Q_u^B}^2 \}$$

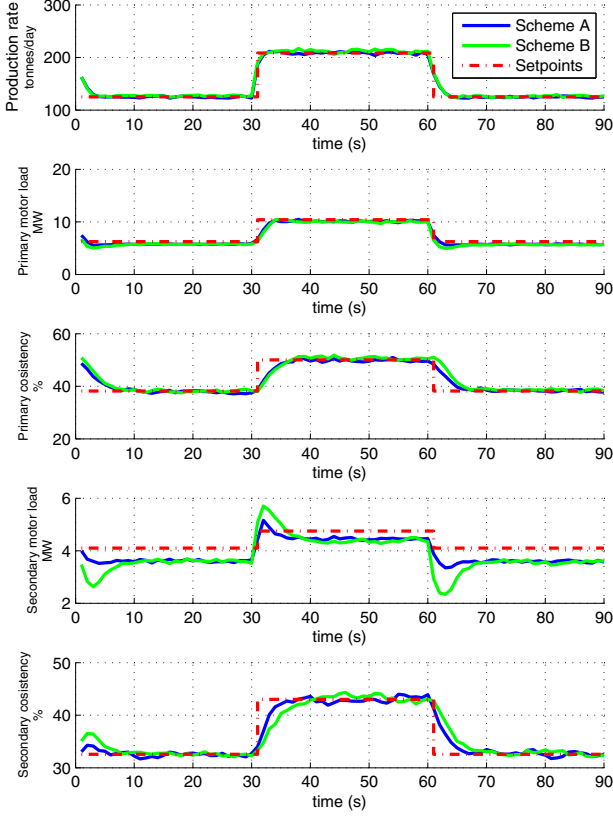


Fig. 3: The state variables of the MP process by using *econ* MPC in Simulation I

$$\begin{aligned}
 &= \min_{(X,U)} \alpha \sum_{i=1}^N \{ (x_2(k+i|k) + x_4(k+i|k)) / x_1(k+i|k) \} \\
 &+ \beta \{ \|x(k+i|k) - x_s(k+i)\|_{Q_x^B}^2 + \|u(k+i|k) - u_s(k+i)\|_{Q_u^B}^2 \}, \quad (17)
 \end{aligned}$$

where  $u_s(k)$  is the steady-state input which can also be obtained from the static optimization as  $x_s(k)$  in scheme A.  $Q_x^B$  and  $Q_u^B$  are the corresponding weights in scheme B. To have a fair comparison between scheme A and B, we specify  $Q_x^B = Q_x^A$  and  $Q_u^B = Q_{\Delta u}^A$ . The optimization constraints and the other variables and parameters in Scheme B are chosen to be the same as those of scheme A.

In the closed-loop simulation, in order to demonstrate the economical benefits of the proposed *econ* MPC with different penalty methods, we also considered the disturbances in the chips in the state variables, as shown in equation (1). The disturbances in the refining process normally arise from the wood chip raw material and the refining process itself. The disturbances will affect the process operating conditions, thereby changing the final pulp quality. Fig. 2 and Fig. 3 demonstrate the pulp properties (CSF, LFC, and SC) after the two-stage refining process and the state tracking performance using *econ* MPC. As illustrated in Fig. 2 and Fig. 3, the *econ* MPC with both the penalty methods in Scheme A and Scheme B are able to ensure the pulp qualities to be

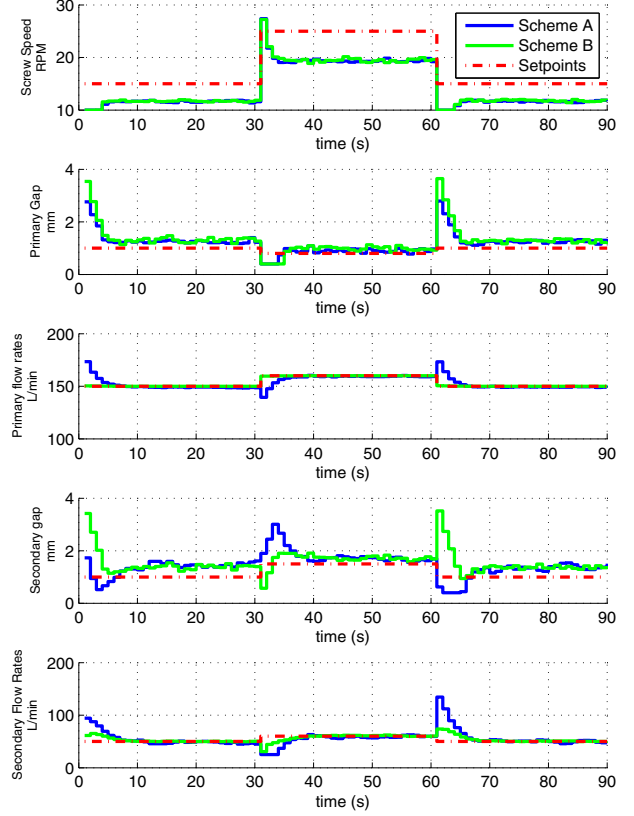


Fig. 4: The manipulated variables of the MP process by using *econ* MPC in Simulation I

within their quality bounds which are 0 ~ 400ml, 0 ~ 100%, and 0 ~ 4% for CSF, LFC, and SC, respectively. Besides, the two schemes demonstrate a similar tracking performance of the state variables. In Fig. 4, it can be seen that a better tracking performance can be achieved by using *econ* MPC with the penalty in Scheme B. However, in Scheme A, instead of emphasizing the tracking effect of the manipulated variable, we only penalize the increment of the input, which as a consequence, will give the nonlinear MPC controller leeway to find its optimal input that minimizes the cost function. The specific energy plot in Fig. 5 further supports our analysis, in which 3.54% of the specific energy reduction is achieved by using Scheme A.

#### B. Simulation II: Energy reduction by using economic MPC compared with standard MPC

In the standard MPC, the optimization problem, which is subject to constraints in (16), is defined in the following form,

$$\begin{aligned}
 V_N^{spt}(x) := \min_{(X,U)} \sum_{i=1}^N \{ &\|x(k+i|k) - x_s(k+i)\|_{Q_x^{spt}}^2 \\
 &+ \|u(k+i|k) - u_s(k+i)\|_{Q_u^{spt}}^2 \}, \quad (18)
 \end{aligned}$$

where  $Q_x^{spt}$  and  $Q_u^{spt}$  are weighting matrices. In this simulation, a potential 18% of the specific energy reduction by

using *econ* MPC in Scheme A, and about 14% of the energy reduction can be achieved by using Scheme B. A more detailed work about the energy reduction by using nonlinear MPC in comparison with the standard MPC can be found in [16]. Note that in both simulations, the computation time of solving each optimization using IPOPT ranges between 0.04–0.09 s, which is much faster than the *fmincon*.

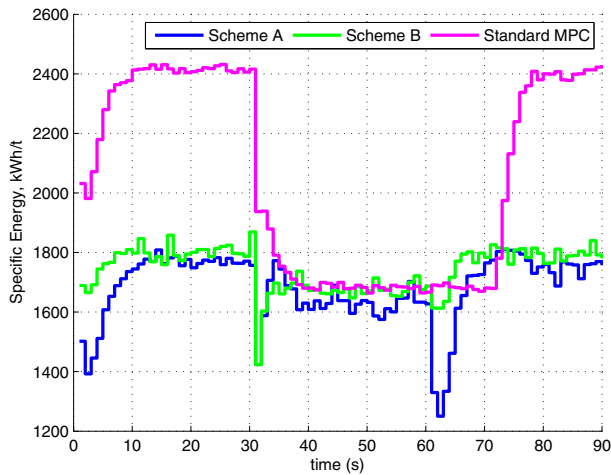


Fig. 5: Comparison of the energy reduction in Simulation I & II

## V. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed the *econ* MPC technique for a two-stage MP process. Two different schemes were studied to guarantee the convergence and stability of the closed-loop nonlinear MP process. In Scheme A, two convex terms, the deviation of the state from its target and the increment of manipulated variables, were added in the objective function. In Scheme B, the deviation of both the state and manipulated variables were included in the cost function.

However, in the simulation results, it can be also observed that some of the set-points, for instance, the secondary motor load (in Fig. 3) and the screw speed (in Fig. 4), are not reached. The controllers settle at a steady-state that is close to the desired one. The authors are now looking into a better balance between tracking and economic without comprising the asymptotic stability. In the recent work [22], the author incorporated a stabilizing constraint to the *econ* MPC that preserves stability of the auxiliary MPC controller. Based on this new result, our future work would be applying the multiobjective optimization methods to the MP process that can not only reduce the electrical energy consumption but also guarantee the asymptotic stability and improve the pulp properties.

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