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Multiobjective Economic Model Predictive Control of Mechanical Pulping Processes

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Abstract— **We present a multiobjective economic model predictive control (*m-econ* MPC) strategy to mitigate high electrical energy demands of a two-stage mechanical pulping (MP) process. The nonlinear MP process considered in this paper consists of a primary and a secondary refiner. In the proposed *m-econ* MPC technique, economic performance is optimized during dynamic transitions while a stabilizing constraint is used to ensure convergence to steady-state operating points. Through simulations, we demonstrate that the *m-econ* MPC can reduce the energy consumed by mechanical pulping processes by up to 27%.**

I. Introduction

The mechanical pulping (MP) processes such as thermo-mechanical pulping (TMP), chemi-thermo-mechanical pulping (CTMP) and bio-chemi-thermo-mechanical pulping (BCMP) play a dominant role in the pulp and paper industry. However, for the past several years this industry has been facing a number of economic challenges due to the increasing price of electricity, changing demands of product markets, and the stiff competition from low cost producers [1]–[3] around the world. With these challenges, the MP mills are seeking novel advanced process control techniques to adapt to the new product production lines and to produce higher quality pulp while simultaneously minimizing the energy consumption.

In the past few years a number of *econ* MPC algorithms have been proposed to reduce the electrical energy usage [4]–[7]. The *econ* MPC is particularly difficult to apply on the MP processes due to their severe nonlinearity and the conflicting objectives in tracking performance and in minimizing energy costs. Clearly, exceptional tracking performance requires high energy input and vice versa. Therefore the ultimate goal of an *econ* MPC on this process is to minimize the energy costs while ensuring that the product meets the minimum quality requirements. In many cases, direct minimization of the energy costs results in an unstable closed-loop MP system. This motivates us to develop advanced control techniques that not only reduce the electrical energy consumption for the MP mills but also guarantee the asymptotic stability of the closed-loop MP system and meet the pulp property requirements.

Stability of economic MPC (*econ* MPC) has been a subject of active research in the last few years. It is now known that stability of general nonlinear *econ* MPC formulations cannot be established using a traditional Lyapunov setting. One strategy to circumvent this problem includes the addition of regularization (convexification) terms guaranteeing that the regularized economic value function becomes a Lyapunov function, as is done in [8], [9]. An alternate approach is to rely on system-theoretic properties (e.g., dissipativity) to guarantee stability; this approach is discussed in [10]. Recently, Zavala *et al* [11] interpreted the *econ* MPC as a multiobjective optimization problem that seeks to simultaneously minimize economic and tracking performance. This interpretation allowed the author to construct a stabilizing constraint that guarantees closed-loop stability for general economic MPC formulations.

In this paper, we address the stability problem of the *econ* MPC problem for a nonlinear two-stage mechanical pulping process by developing the multiobjective *m-econ* MPC algorithm. The remainder of this paper is divided as follows: in Section II, we develop a nonlinear model for the two-stage refining process, in Section III, the *m-econ* MPC technique for the two-stage MP process is presented, and in Section IV, simulation studies examining the performance of the proposed *m-econ* MPC technique are provided followed by conclusions in Section V.

II. Two-stage High Consistency Refining Process

A. Process Description

A mechanical pulping process generally involves three operational units: wood chip pretreatment, wood chip refining and pulp processing as shown in Fig. 1. In these operating units, the wood chips are broken down into individual fibres in the refiners. Typically, there are two high consistency (HC) refiners - a primary and a secondary refiner. The pulp consistency is defined as the ratio of mass of dry fibres to the total mass of dry fibre and water suspension. When this ratio is between 20% and 50%, it is called high consistency (HC) and when it is between 3% and 5% it is called low consistency (LC). These refiners play a vital role as they are the most energy consuming units in the entire process. As such controlling and optimizing the performance of the refiners is essential to the economic viability of these mills.

B. Two-stage HC refining model

The dynamic models of mechanical refining process have been developed and reported recently in [4]–[7]. The authors in [4] and [5] proposed a model for a two-stage HC refining process. In [6], a multi-stage refining process model, which consists of both HC and LC refiners, was studied. The model

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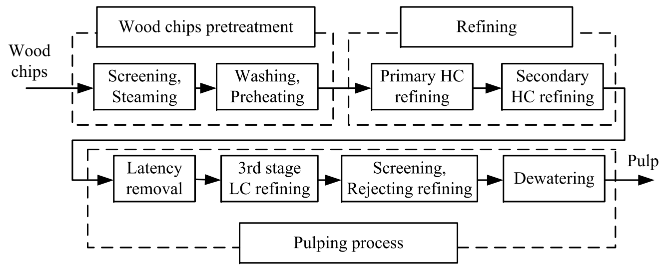

Fig. 1: Operational units in a typical two-stage MP process

employed in this paper will be based on the models derived in our previous work [7]. The key variables required for control and optimization of the process are shown in Fig. 2.

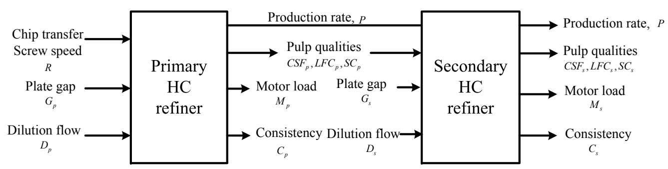

Fig. 2: The process variables for the two-stage MP process

1) Production rate: In a MP process, the production rate P ($tonnes/day$) can be characterized by the wood flow fed to the refiner. In this paper, the following production rate model will be used [7],

$$P = k_a \cdot k_p \cdot s_c \cdot d_c \cdot R, \tag{1}$$

where k_a and k_p (m^3/rev) are constant parameters which can be obtained from the industrial data and their values depend on the particular production lines. $s_c(\%)$ is the chip solid content. d_c (kg/m^3) is the chip bulk density. R (rpm) is the chip-transfer screw speed.

2) Motor load: The motor load is a function of the production rate, refiner gap size, and the dilution flow rate [7]. The motor load model is defined as follows,

$$M_i = \frac{k_{m_i} \cdot P}{D_i}(1 - e^{(-10G_i)})(a_i - b_i \cdot G_i), i = 1, 2, \tag{2}$$

where M_i (MW) is the motor load for the i-th refiner, $i = 1, 2$. D_i (l/min) is the dilution water flow rate. G_i (mm) is the gap distance. a_i, b_i, and k_{m_i} are the parameters of each refiner.

3) Consistency: The refining consistency plays an important role in determining the final pulp quality. However, due to the lack of online sensors, it is difficult to measure the consistency in the refining zone of a plate. In this work, we assume that the consistency can be modeled as follows [6],

$$C_p = \frac{100P}{P + k_a \cdot D_p - k_{e_p} \cdot M_p}, \tag{3}$$

$$C_s = \frac{100P}{P/(0.01C_p) + k_a \cdot D_s - k_{e_s} \cdot M_s}, \tag{4}$$

where C_p and C_s are the consistency after the primary and secondary refiner, respectively. k_a, k_{e_p} and k_{e_s} are the refiner parameters.

4) Specific energy (S): In this process, the specific energy (MW/tonnes/day) is one of the main variables that has a significant effect on the pulp properties [3]. The majority of the literature on refiner optimization, cost reduction and product quality improvement is based on minimizing the specific energy consumed by the refiners [4]–[7]. The specific energy can be interpreted as the energy applied on the chips to convert them into fibres. In this paper, we define specific energy S_t to be the total motor load over the production rate at time instance t,

$$\text{Specific Energy (S)} := \frac{\text{Total motor load}}{\text{Production rate}}. \tag{5}$$

5) Disturbances: The majority of the disturbance variables in a MP process are the variables that cannot be controlled or even be measured. However, these disturbances affect the refining conditions and thus influence the final pulp quality. In this paper, the variations in raw material, specifically, the chip bulk density and chip moisture content, are considered to be the main disturbances.

6) Pulp properties: A number of pulp properties are used to characterize the refined pulp. In this paper, the properties of Canadian Standard Freeness (CSF, *ml*), the long fibre content (LFC, %) and the shive content (SC, %) are used to assess the quality of the pulp, as they are amongst the most commonly used properties to describe the quality of mechanical pulp.

The production rate, motor loads and consistencies for both primary and secondary refiners are treated as the discretized differential state variables while the pulp properties after each refiner are treated as the algebraic state variables. The chip-transfer screw speed, plate gap and dilution water flow rates of each refiner are taken as the manipulated variables. The linear dynamics of the discretized differential state variables and disturbances are modeled using data derived from several identification experiments on industrial processes. By superimposing the linear dynamics with the steady-state model (1)-(5), the discrete-time nonlinear model for the two-stage MP process at time instant t can be written as,

$$x_{t+1} = Ax_t + h(x_t, u_t) = f(x_t, u_t), \tag{6}$$

$$0 = g(x_t, y_t), \tag{7}$$

where $x_t \in \mathbb{R}^{n_x}$, $u_t \in \mathbb{R}^{n_u}$, and $y_t \in \mathbb{R}^{n_y}$ represent the state variables, the manipulated variables, and the controlled outputs respectively. x_t, y_t and u_t are defined in (8) below. $A \in \mathbb{R}^{n_x \times n_x}$ is the dynamic matrix. $h(\cdot) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \mapsto \mathbb{R}^{n_x}$ is the nonlinear function which maps the input and state to the dynamic variable in the differential state of the MP process. The state and input variables are required to satisfy the constraint $x_t \in \mathbb{X}$ and $u_t \in \mathbb{U}$. The sets $\mathbb{X} \subseteq \mathbb{R}^{n_x}$ and $\mathbb{U} \subseteq \mathbb{R}^{n_u}$ are compact and contain the equilibrium point (x_s, u_s). The mapping $f(\cdot) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \mapsto \mathbb{R}^{n_x}$ is assumed to satisfy $f(x_s, u_s) = x_s$ at the equilibrium point. $g(\cdot)$ is an implicit nonlinear function between the refining condition and the pulp properties.

$$x_t = \begin{bmatrix} \text{Production rate, } P \\ \text{Primary motor load, } M_p \\ \text{Primary consistency, } C_p \\ \text{Secondary motor load, } M_s \\ \text{Secondary consistency, } C_s \end{bmatrix},$$

$$y_t = \begin{bmatrix} \text{CSF after primary refining, } CSF_p \\ \text{LFC after primary refining, } LFC_p \\ \text{SC after primary refining, } SC_p \\ \text{CSF after secondary refining, } CSF_s \\ \text{LFC after secondary refining, } LFC_s \\ \text{SC after secondary refining, } SC_s \end{bmatrix},$$

$$u_t = \begin{bmatrix} \text{Chip-transfer screw speed, } R \\ \text{Primary refiner plate gap, } G_p \\ \text{Primary dilution flow rate, } D_p \\ \text{Secondary refiner plate gap, } G_s \\ \text{Secondary dilution flow rate, } D_s \end{bmatrix}. \quad (8)$$

III. Multiobjective economic model predictive control for the MP Process

A. Basic Notation

Following the notation used in [11] the set point tracking value function V_t^{tr} and the economic value function V_t^{ec}, at time t, are,

$$V_t^{tr} := \sum_{k=t}^{t+T-1} L^{tr}(x_k - x_s, u_k - u_s), \quad (9)$$

$$V_t^{ec} := \sum_{k=t}^{t+T-1} L^{ec}(x_k, u_k), \quad (10)$$

where L^{tr} and L^{ec} are the tracking stage cost and economic stage cost, respectively. The mapping $L^{tr} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \mapsto \mathbb{R}$ is always nonnegative, and $L^{tr} = 0$ holds if and only if $x_t = x_s$ and $u_t = u_s$. In the sequel we will use $L^{tr}(x_k, u_k)$ as a compact form of the stage cost $L^{tr}(x_k - x_s, u_k - u_s)$. The economic stage cost $L^{ec} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \mapsto \mathbb{R}$ is bounded and related directly to desired economics (not necessarily dependent on the steady-state (x_s, u_s)). For brevity, the trajectory of x_k, $k = t, \ldots, t+T$, and u_k, $k = t, \ldots, t+T-1$, will be denoted by $\{x_k, u_k\}_t^{t+T}$ in the rest of this paper.

In the MPC literature, the *admissible set* $\mathbb{W}_T$ in $T+1$ steps is defined as a joint set comprised of the initial states x_0 and the feasible input sequence $\mathbf{u_T} = (u_0, u_1, \ldots, u_{T-1})$. To be specific,

$$\mathbb{W}_T := \{(x_0, \mathbf{u}) | \exists\ x_1, \ldots, x_T : x_{t+1} = f(x_t, u_t), \\ x_t \in \mathbb{X}, u_t \in \mathbb{U}, \text{and } x_T = x_s\}. \quad (11)$$

The corresponding set of *admissible initial states* $\mathbb{Z}_T$ is then defined as the projection of $\mathbb{W}_T$,

$$\mathbb{Z}_T := \{x_0 | \exists\ \mathbf{u}_T : (x_0, \mathbf{u}_T) \in \mathbb{W}_T\}. \quad (12)$$

Assume that at time instant $t+1$, the optimal trajectory $\{\bar{x}_{k|t+1}, \bar{u}_{k|t+1}\}_{t+1}^{t+1+T}$ is the solution of the following tracking MPC (*tr* MPC) problem,

$$\begin{aligned} \min_{z_k, v_k} \quad & \sum_{k=0}^{T-1} L^{tr}(z_k, v_k), \\ s.t. \quad & z_{k+1} = f(z_k, v_k), \\ & z_k \in \mathbb{X}, v_k \in \mathbb{U}, \\ & z_0 = x_{t+1}, \\ & z_T = x_s, k = 0, \ldots, T-1, \end{aligned} \quad (13)$$

where z_k, v_k are the optimization variables representing x_k and u_k, respectively.

Remark 3.1. As pointed out in [11] and [12], in the traditional *tr* MPC, the stage cost L^{tr} is a positive definite function. The stability thus can be proved by treating the tracking stage cost as a Lyapunov function. However, the stability cannot be guaranteed when the stage cost is replaced by an arbitrary economic cost L^{ec}. This is because the economic cost may not be a positive definite function, and thus it is not qualified to be a Lyapunov function. For more results on the convergence and stability of the *econ* MPC, see [10], [12]–[14]. Under the strong duality assumption, the authors in [12] propose a class of MPC schemes using an economic cost objective that admits a Lyapunov function to establish the asymptotic stability properties of the closed-loop system. In [10], a terminal state constraint is used to force the predicted states to converge to the steady states at the end of the horizon. This result is further relaxed in [13] to an asymptotic time-averaged economic MPC without terminal constraints, in which an approximate optimal performance is obtained based on certain controllability assumptions and on the turnpike property. A terminal region constraint is proposed in [14], and the stability is guaranteed by adding a penalty to the terminal state in the cost function.

B. Multiobjective MPC Controller Design

In the standard *tr* MPC technique, $\{\bar{x}_{k|t+1}, \bar{u}_{k|t+1}\}_{t+1}^{t+1+T}$ is the optimal trajectory. As explained above, the stability of the closed-loop system can be achieved by treating the tracking stage cost L^{tr} as a Lyapunov function. Moreover, it has been shown that stability can be guaranteed for any MPC controller generating a feasible trajectory $\{x_{k|t+1}, u_{k|t+1}\}_{t+1}^{t+1+T}$ as long as such trajectory satisfies:

$$V_{t+1}^{tr} \leq \bar{V}_{t+1}^{tr} + \sigma(V_t^{tr} - \bar{V}_{t+1}^{tr}), \quad (14)$$

where $\sigma \in [0,1)$ is a scalar. $\bar{V}_t^{tr} := \sum_{k=t+1}^{t+T} L^{tr}(\bar{x}_{k|t+1}, \bar{u}_{k|t+1})$ is the value function of the optimal trajectory $\{\bar{x}_{k|t+1}, \bar{u}_{k|t+1}\}_{t+1}^{t+1+T}$ for the tracking MPC at time instant $t+1$. The condition (14) is also referred as the *stabilizing constraint*. The function V_{t+1}^{tr} is defined as,

$$V_{t+1}^{tr} := \sum_{k=t+1}^{t+T} L^{tr}(x_{k|t+1}, u_{k|t+1}). \quad (15)$$

Because any feasible trajectory satisfying (14) is guaranteed to lead to stability, we can design an *econ* MPC controller that enforces (14) directly. The formulation of this controller is given by:

$$\min_{z_k,v_k} \sum_{k=0}^{T-1} L^{ec}(z_k,v_k), \tag{16}$$
$$\begin{aligned} s.t.\quad & z_0 = x_{t+1}, z_T = x_s, \\ & z_{k+1} = f(z_k,v_k), \\ & z_k \in \mathbb{X}, v_k \in \mathbb{U}, \\ & \sum_{k=0}^{T-1} L^{tr}(z_k,v_k) \le \varepsilon_{t+1}(\sigma), \quad (17) \\ & k = 0,\dots,T-1, \end{aligned}$$

where

$$\varepsilon_{t+1}(\sigma) := \bar{V}^{tr}_{t+1} + \sigma(V^{tr}_t - \bar{V}^{tr}_{t+1}). \tag{18}$$

Notice that the constraint in (17) is an equivalent expression of the stabilizing constraint in (14). The specific algorithm we propose to solve the above *m-econ* MPC problem is illustrated in Table I.

TABLE I: The implementation of *m-econ* MPC

Algorithm	of the *m-econ* MPC
Input:	$x_0 \in \mathbb{X}$, $\sigma \in [0,1)$, set $t \leftarrow 0$ and $\varepsilon_0(\sigma) \leftarrow +\infty$.
Output:	**for** $t = 0,\dots,$ simulation ends **do**
1:	Solve the *m-econ* MPC optimization in (16)-(18) for the state x_t and $\varepsilon_t(\sigma)$, evaluate V^{tr}_t, and set $u_t \leftarrow v_0$.
2:	Implement u_t to the plant and obtain the state variables $x_{t+1} = f(x_t,u_t)$.
3:	Solve *tr* MPC in (13) for the state x_{t+1}, and evaluate $\bar{V}^{tr}_{t+1}$.
4:	Set $\varepsilon_{t+1}(\sigma) \leftarrow \bar{V}^{tr}_{t+1} + \sigma(V^{tr}_t - \bar{V}^{tr}_{t+1})$.
5:	**end for**

Recall that in the *m-econ* MPC technique, the closed-loop stability is implied by the stabilizing constraint in (14). The proof of this statement is outlined as follows.

Assume that the trajectory $\{\bar{x}_{k|t+1}, \bar{u}_{k|t+1}\}^{t+1+T}_{t+1}$ is an optimal trajectory for the standard *tr* MPC problem in (13). Then the following inequality holds [15],

$$\bar{V}^{tr}_{t+1} - V^{tr}_t \le -L^{tr}(x_t,u_t). \tag{19}$$

Adding the term $-V^{tr}_t$ to both sides of (14), we have,

$$V^{tr}_{t+1} - V^{tr}_t \le (1-\sigma)(\bar{V}^{tr}_{t+1} - V^{tr}_t). \tag{20}$$

Combining (19) and (20), the following inequality holds for any feasible solution,

$$V^{tr}_{t+1} - V^{tr}_t \le -(1-\sigma)L^{tr}(x_t,u_t). \tag{21}$$

Since the function $(1-\sigma)L^{tr}(x_t,u_t)$ is nonnegative for $\sigma \in [0,1)$, we can conclude that the trajectory $\{x_{k|t+1}, u_{k|t+1}\}^{t+1+T}_{t+1}$ obtained from the *m-econ* MPC algorithm is stable. It has been proved in [11] that the asymptotically stablility with the region of attraction $\mathbb{Z}_T$ can be guaranteed under the the control law obtained by solving the *m-econ* MPC problems for any $\sigma \in [0,1)$.

C. Multiobjective Economic MPC for the MP Process

The MP process is a complex multi-input multi-output (MIMO) nonlinear process with strong interactions among the variables. Based on the MP model developed in (6)-(7) and the *m-econ* MPC approach proposed in the previous subsection, we are now in a position to apply the *m-econ* MPC technique to the MP process. For the MP process, the tracking and economic objective functions are defined to be,

$$V^{tr}_t = \sum_{k=t}^{t+T-1} ||x_k - x_s||^2_{Q_x} + ||u_k - u_s||^2_{Q_u}, \tag{22}$$
$$V^{ec}_t = \sum_{k=t}^{t+T-1} S_k, \tag{23}$$

where T is the prediction horizon, Q_x and Q_u are positive-definite weighting matrices for the state and input variables, respectively. S_k is the specific energy as defined in (5), which is $(x_k(2)+x_k(4))/x_k(1)$ in our MP process. In this work, all the state variables in the MP process are assumed to be available at each sampling time t. The *m-econ* MPC optimization for the MP process can be formulated as follows,

$$\begin{aligned} \min_{z_k,v_k}\quad & \sum_{k=t}^{t+T-1} S_k, \\ s.t.\quad & z_0 = x_{t+1},\ z_T = x_s, \\ & z_{k+1} = f(z_k,v_k),\ 0 = g(z_k,v_k), \\ & x_{min} \le z_k \le x_{max}, \\ & y_{min} \le y_k \le y_{max}, \\ & u_{min} \le v_k \le u_{max}, \\ & \sum_{k=t}^{t+T-1} ||z_k - x_s||^2_{Q_x} + ||v_k - u_s||^2_{Q_u} \le \varepsilon_{t+1}(\sigma), \\ & k = 0,\dots,T-1, \end{aligned}$$

where $\sigma \in [0,1)$. $\varepsilon_{t+1}(\sigma)$ is defined in (18). x_{min} and x_{max}, y_{min} and y_{max}, u_{min} and u_{max}, are the lower bounds and upper bounds of the states, pulp qualities, the manipulated variables, respectively.

Remark 3.2. In our previous work [7], we proposed an *econ* MPC algorithm for the MP process, in which two different *econ* MPC schemes were investigated: one with penalty on the increment of the input and one with penalty on the offset of the input from its steady-state. It has been observed that in order to reduce the energy consumption in terms of the specific energy, larger penalty has to be added to the economic term in the objective function. However, this may lead to a significant deviation of the state variables from the steady-state target. Similar issues have also been reported in [4]–[6]. This motivates our proposition of using the *m-econ* MPC to solve such problems and achieve an acceptable compromise between the tracking performance and the economics.

IV. Simulation Results

The objective of this simulation is to show the effectiveness and the economical benefits of using the *m-econ* MPC algorithm in the two-stage MP process. To be specific, we will show that the proposed *m-econ* MPC algorithm can not only reduce the energy consumption in terms of specific energy, but also guarantee the closed-loop stability. This algorithm also allows the user to tune the controller in such a way that a desired trade-off between the economic and the tracking performances is achieved based on the practical demands.

TABLE II: The variations of chip bulk density d_c and chip solid content s_c of the raw material from their nominal values

Time (s)	0-50s	50-110s	110-160s
Chip bulk density (d_c)	80%	115%	90%
Chip solid content (s_c)	90%	100%	110%

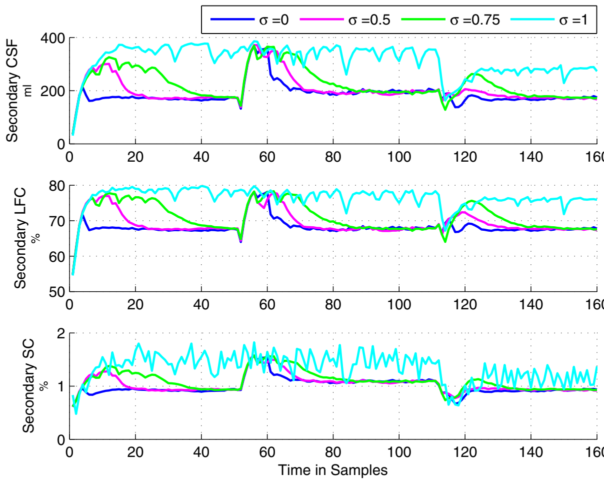

Fig. 3: Pulp qualities after two-stage HC refining

In the closed-loop simulation, the variations in the raw materials such as the chip bulk density d_c and the chip solid content s_c are considered as the disturbances (see Table II). The prediction horizon and the control horizon are selected to be equal and set to be $T = 30$. The sampling time is $2s$, and the simulation length is 160. The weighting matrices $Q_x = diag([0.01, 10, 0.1, 10, 0.1]^T)$ and $Q_u = diag([0.1, 100, 0.01, 100, 0.01]^T)$. As mentioned in Section II, the scalar σ has to be in the range $[0, 1)$ for closed-loop stability. However, to demonstrate the effect of the parameter σ on the tracking performance and the economics, here we allow $\sigma = 1$ and will examine the following four different values of σ: $\sigma = 0$, $\sigma = 0.5$, $\sigma = 0.75$, and $\sigma = 1$. Note that for $\sigma = 0$, the multiobjective economic MPC will be reduced to the standard tracking MPC. When $\sigma = 1$, it will be equivalent to the *econ* MPC without regulations. $\sigma = 0.5$ and $\sigma = 0.75$ are the two cases where we have the standard *m-econ* MPC. To address the computational issues, in this simulation the nonlinear MP process model is built in AMPL (A Mathematical Programming Language), and the nonlinear optimization problem is solved using IPOPT (Interior Point Optimizer) [16].

The simulation results are shown in Fig. 3–6. From Fig. 3, we can see that for these four situations, all of the pulp qualities such as the CSF, LFC, and SC, remain within their respective acceptable ranges: $50 - 400ml$, $50 - 80\%$, and $0 - 2\%$. However, by using the *econ* MPC ($\sigma = 1$), the pulp qualities are more likely to hit the operating limits compared with the other two MPC schemes. This is not desirable from the perspective of mechanical pulping mills. Fig. 4 and Fig. 5 illustrate the tracking performance of the state variables and manipulated variables. It can be seen that for $\sigma = 0$, $\sigma = 0.5$, and $\sigma = 0.75$, the state variables and the manipulated variables converge to the steady state but with different convergence speed. Specifically, as σ decreases, the tracking speed of the *m-econ* MPC improves, which is consistent with our analysis since smaller σ values imply more emphasis on the tracking performance. For the extreme case where $\sigma = 1$, the convergence and stability cannot be guaranteed since the target in this case will be merely achieving the optimal economic performance regardless of the tracking performance or even the stability.

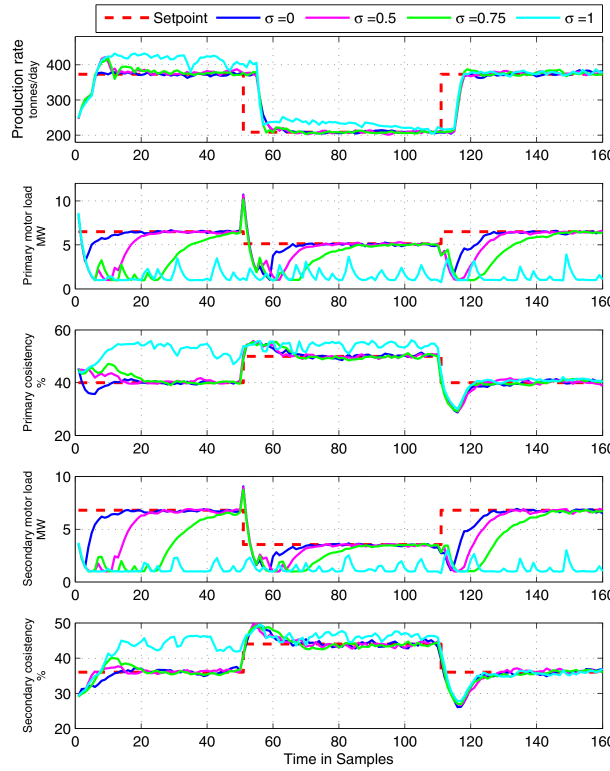

Fig. 4: The state variables of the MP process

The comparison of the specific energy consumption between these four situations is illustrated in Fig. 6. From Fig. 6, we can see that the *m-econ* MPC with $\sigma = 0.5$ and $\sigma = 0.75$ can save about 10% and 27% of the specific energy, respectively, compared with the tracking MPC when $\sigma = 0$. Moreover, the *econ* MPC scheme where $\sigma = 1$ shows the most amount of energy saved at about 73.12% but the closed-loop MP process is unstable.

V. Conclusion

We presented an multiobjective economic (*m-econ*) MPC technique for a two-stage MP process. We show that this ap-

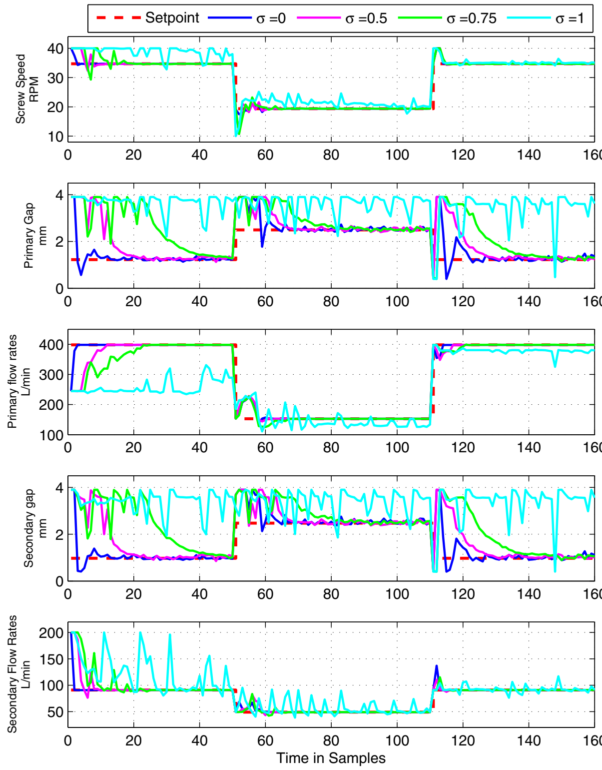

Fig. 5: The manipulated variables of the MP process

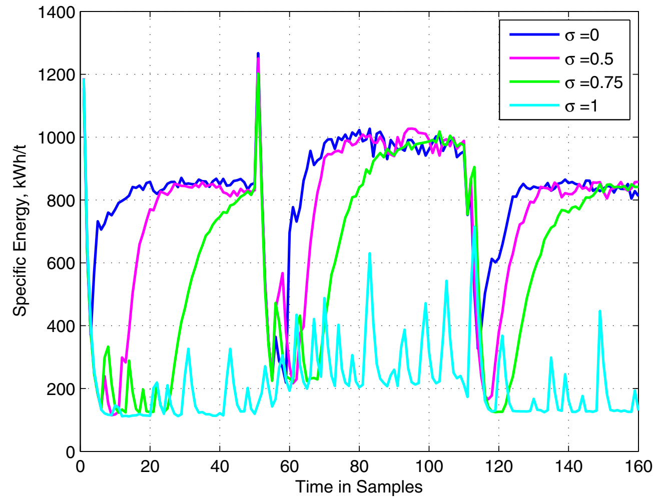

Fig. 6: Comparison of the energy reduction

proach leads to drastic reductions in energy demands for MP processes while stable operations are ensured by exploiting a stabilizing constraint. The proposed *m-econ* MPC controller enables to trade-off economics and the convergence rate to the equilibrium set-point.

VI. ACKNOWLEDGMENTS

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