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A Chance-Constrained Nonlinear Programming Approach for Equipment Design Under Uncertainty

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Abstract

In this work there are shown different strategies to cope uncertainty in large-scale chance-constrained nonlinear programs. We present the design of a flare system as a case study. The design of this system is influenced by several uncertain factors, such as the volume and composition of the waste flow stream to be combusted and the ambient conditions. These systems are currently designed based on typical historical values for waste fuel gases and ambient conditions. Consequently, an improperly designed flare can be susceptible to extreme events previously not experienced. Particularly, we use moment matching (MM) when the algebraic form of the moments and the quantile function of the chance constrained (CC) distribution is known, and for more general settings when the distribution cannot be predicted we use the scenario approach (AS), the popular conditional value at risk (CVaR) and the recently proposed sigmoid value at risk (SigVaR). We demonstrate that the SigVaR approximation offers the best results and this approach overcome the conservative results of the AS and CVaR.

Keywords: Uncertainty, design, sigmoid conditional value at risk, flares.

1. Introduction

We study the chance-constrained nonlinear program:

$$\min_{d \in D} \varphi(d) \tag{1a}$$

Subject to

$$\mathbb{P}\left(f\left(d,\Xi\right) \leq \overline{f}\right) \geq 1-\alpha \tag{1b}$$

We review exact approaches to handle chance constrained problems (CC-P). We consider the special case in which the algebraic form of the quantile function is known or approximately known, and we propose to use moment matching to compute its parameters. However, there are some cases where the nonlinear chance constrained problems are particularly difficult to solve because nonlinear propagation makes it hard to obtain the distribution of output variables (this case will be analysed in a future work).

To handle more general settings, we consider the use of three different approximations. The first one is the scenario approximation, in this approach we approximate the chance constrain ensuring that the constrain is satisfied for each scenario or with probability of 1, and this implies that the constraint is satisfied for any probability different to one. This is extremely conservative; nevertheless, it let us express the chance constrain using the standard scenario-based stochastic programming formulation. The second approximation is the conditional value at risk, this approximation enables expressing our problem as a common nonlinear optimization problem, but CVaR can be slightly conservative. Moreover, the CVaR approximation does not offer a mechanism to enforce convergence to a solution of CC. The third approximation is the sigmoid value at risk approximation (Cao and Zavala, 2018), which provides a mechanism to determine exact solutions for CC-P.

2. Flare systems

Gas flares are used as safety (relief) devices that are used all over the world to manage abnormal situations in infrastructure systems (natural gas and oil processing plants and pipelines), manufacturing facilities (chemical plants, offshore rigs), and power generation facilities. Abnormal situations include equipment failures, off-specification products, and excess materials in start-up/shutdown procedures. In particular, flares prevent over pressurization of equipment and use combustion to convert flammable, toxic or corrosive vapors to less-dangerous compounds (Sorrels et al., 2017). A proper design of flare systems is vitally important due to it is influenced by several uncertain factors, such as the amount and composition of the waste fuel gases to be combusted and the ambient conditions like wind velocity. An improperly designed flare can be susceptible to extreme events previously not experienced or have as a result an oversized and expensive equipment. Here, it is proposed to use stochastic programming formulations to systematically capture uncertain conditions in the design procedure.

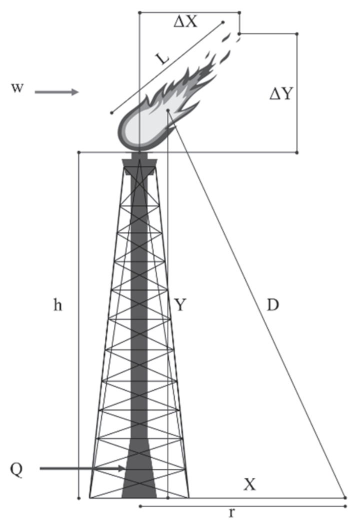

Figure 1. Flare system generic diagram.

3. Physical model

It was formulated a mathematical model for sizing a flare system (see Figure 1). The design goals are to minimize the equipment cost while controlling the thermal radiation level at ground level. It was based on the American Petroleum Institute standard 521 as follows:

The heat released by combustion H (BTU/h) is a function of the random input waste flow Q (lb/h) and the heat of combustion h_c (BTU/lb):

$$H = h_c Q \tag{1}$$

The flame length L (ft) can be calculated as a function of the released heat using an approximation of the form:

$$\log L = a_1 \log H - a_2 \tag{2}$$

The wind speed w (ft/s) is an important environmental factor that affects the tilting of the flame and the distance from the center of the flame. The following correlations capture the flame distortion as a result of the wind speed and the exit velocity:

$$\log \Delta X = \log(a_5 L) + a_6 (\log w - \log U) \tag{3}$$

$$\log \Delta Y = \log(a_7 L) - a_8 (\log w - \log U) \tag{4}$$

The flare stack diameter t (ft) is sized on a velocity basis. This is done by relating this to the Mach number M and the waste flow as:

$$M^2 = \left(\frac{a_3}{t^2}\right) Q^2 \tag{5}$$

The flare tip exit velocity U (ft/s) is function of the flow and the diameter:

$$U = a_4 \frac{Q}{t^2} \tag{6}$$

Here, ΔX and ΔY (ft) are the horizontal and vertical distortions. The distortions are used to compute the horizontal X, vertical Y, and total distance D (ft) to a given ground-level safe point (r,0) as:

$$X = r - \left(\frac{1}{2}\right)\Delta X \tag{7}$$

$$Y = h + \left(\frac{1}{2}\right)\Delta Y \tag{8}$$

$$D^2 = X^2 + Y^2 \tag{9}$$

Here, h (ft) is the flare height. The flame radiation K (BTU/h ft^2) is a function of the heat released and the total distance:

$$K = a_9\left(\frac{H}{D^2}\right) \tag{10}$$

A primary safety goal in the flare stack design problem is to control the risk that the radiation exceeds a certain threshold value $\overline{k}$ (BTU/h ft^2) at the ground-level reference point (r,0). This is modelled using the CC:

$$\mathbb{P}\left(K \le \overline{k}\right) \ge 1 - \alpha \tag{11}$$

The objective function is the cost (USD), which is a function of height and diameter:

$$\varphi(t,h) = \left(a_{10} + a_{11}\, t + a_{12}\, h\right)^2 \tag{12}$$

The height and the diameter play a key role in controlling the radiation at the reference point (i.e., a higher and wider flare reduces the radiation intensity). As a result, there is an inherent trade-off between capital cost and safety that must be carefully handled.

4. Case study

The design was considered of a flare stack that combusts a waste fuel gas flow, Q (see Figure 1). The goal is to design a flare system that minimizes cost and satisfies CC on the thermal radiation using the AS, CVaR, SigVaR, and MM approaches. In this work, the case is presented in which the input flow to the flare stack follows a log-normal distribution (with mean a = 10,000 lb/h and standard deviation b = 3,000 lb/h to exemplify a distribution with isolated events). All formulations were solved using 1000 random samples for the inlet flow (see Figure 2). The flare design must satisfy a CC on the radiation with a maximum threshold ($\overline{k}$) of 2,000 BTU/h ft^2 and with a probability $1-\alpha = 0.95$. The optimization formulation is an NLP with 9,005 variables that was implemented using the open-source modelling language JuMP (Bezanson et al., 2017) and solved with Ipopt (Wächter and Biegler, 2006).

5. Results

Table 1 presents the optimal cost, diameter, and height for the flare system under the used approaches. The results show that the design of the AS approximation is the most conservative. The design is 64% more expensive than the MM design. This is since the

AS approach does not allow for explicit control of the probability of constraint satisfaction. The CVaR approximation reduces this highly conservatism, but it considerably overdesigns the flare stack. The optimal height obtained with CVaR is 67.70% larger and 20% more expensive than those obtained with MM; therefore, CVaR approximation reduces the extreme conservatism of the AS, but it still overdesigned. Table 1 also presents that the optimal solution of SigVaR ad MM are very close. The SigVaR design is only 1% more expensive than the MM approach, and that let to highlight the fact that the SigVaR solution is rightly conservative.

Table 1. Optimal Values for Design Variables

	AS	**CVaR**	**SigVaR**	**MM**
Cost (USD)	233,179	172,329	143,609	142,128
Diameter (ft)	1.70	1.70	1.70	1.70
Height (ft)	179.87	105.08	65.16	63.00

Figure 2 shows the empirical PDFs for the input flow and for the radiation at the optimal solution of the CVaR, SigVaR, and MM approaches. First, it is observed that, for the MM approach, the distribution of the radiation is indeed log-normal. This indicates that the structure of the flare model preserves the log-normal shape of the input flow. It is also observed that the histograms of SigVaR and MM are quite similar, with SigVaR being slightly more conservative. In particular, the tail of the SigVaR distribution is very similar to that of MM (the tail reaches values of 6,000 BTU/h ft^2 for SigVaR, compared to 6,100 BTU/h ft^2 with MM). The histogram of CVaR further validates the observation that this approach is very conservative (the tail reaches values of 4,000 BTU/h ft^2)

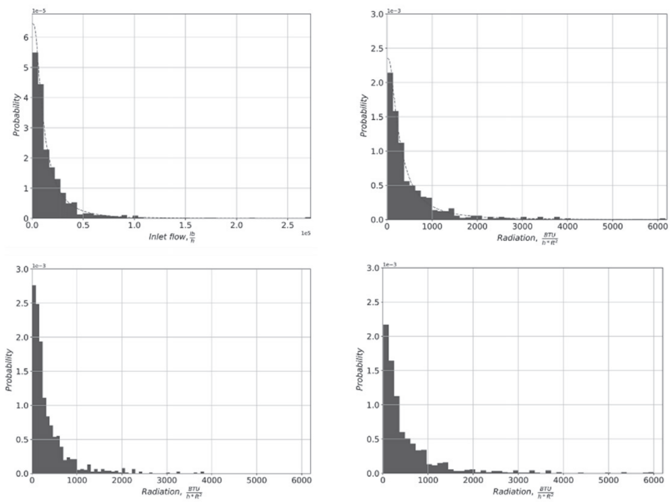

Figure 2. (Top-left) Inlet flow PDF, (top-right) radiation PDF using MM, (bottom-left) radiation PDF using CVaR, and (bottom-right) radiation PDF using SigVaR.

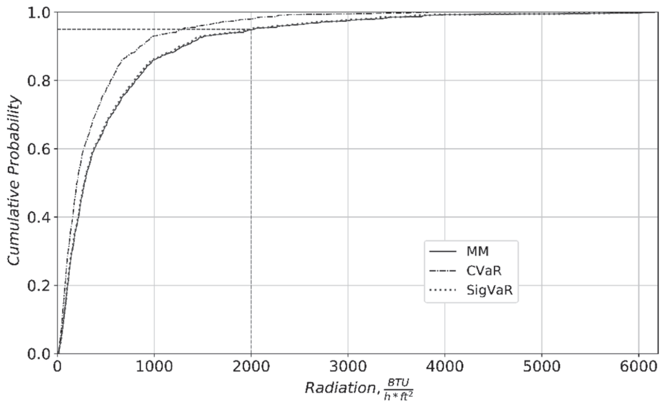

Figure 3. Radiation CDFs using MM, CVaR and SigVaR.

Figure 3 compares the optimal radiation CDFs obtained with CVaR, SigVaR, and MM. It can be seen again that CVaR is very conservative, achieving probability levels for the chance constraint of 98−99% (when 95% is only required). Also, the CDFs for SigVaR and MM overlap for the log-normal case, reinforcing that SigVaR provides good quality approximations and that the distribution of the radiation is indeed log-normal and thus MM is a good approach to solve the problem.

6. Conclusions

An application of moment matching techniques was presented to reformulate chance constraints when the shape of the underlying density function is known. It was also demonstrated the use of conservative approximations, that can be applied to more general setting in which the shape of the density function in unknown. A flare system design study showed that a sigmoid approximation overcomes the conservativeness of the popular conditional value-at-risk and scenario approaches, and it can be an excellent alternative when moment matching is not applicable. The proposed approaches enable the solution of large-scale NLPs with chance constraints.

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