FAIRNESS MEASURES FOR DECISION-MAKING AND CONFLICT RESOLUTION

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Abstract

Allocating utility among stakeholders is a fundamental decision-making task that arises in complex organizations, social planning, infrastructures, and markets. In this work, we review concepts of fairness from the perspectives of game theory, economics, statistics, and engineering by using an axiomatic approach. Our work reveals significant deficiencies in the social welfare allocation approach (which is widely used in the engineering literature) and highlights interesting and desirable properties and connections between Nash and entropy allocation approaches.

Keywords

Fairness, Utility allocation, Optimization, Decision-making.

Introduction

A common measure used in determining utility allocations among stakeholders (representing sub-organizations, subsystems, individuals) that comprise a community (representing enterprises, systems, markets) is the *total utility* (the sum of individual utilities). This allocation paradigm, often known as the classical utilitarian approach or as the *social welfare* approach, is intuitive but might yield allocations that are not fair. The lack of fairness is the result of inherent solution degeneracies (multiple allocations can yield the same total utility) and to the extreme sensitivity of this approach to subsystem scales. For example, in the case of wholesale electricity markets, the market is cleared by solving a maximum total utility allocation problem that determines demand and supply allocations to various stakeholders (utility companies and power producers) (Zavala et al. 2017). Here, allocations tend to favor large stakeholders over small ones and multiplicity of solutions are often encountered when many market participants are present (due to large numbers of degrees of freedom).

In the field of game theory, the utility allocation problem has been viewed as a bargaining game between stakeholders. Nash (1950) first provided an axiomatic approach to obtain fair solutions to the bargaining problem. These axioms include Pareto optimality, symmetry, affine invariance, and independence of irrelevant alternatives. Nash also proved that there exists a utility allocation scheme that satisfies these axioms (what is now known as the Nash solution). A generalization of Nash's scheme is the proportional fairness scheme, which has been widely used to allocate bandwidth in telecommunication networks (Zukerman et al. 2005). Fairness measures have also been widely used to quantify income inequality (Venkatasubramanian 2017). For instance, the Gini coefficient is often used to rank nations according to prevalent income inequality and to quantify the impacts of various economic policies to reshape the income distribution (Pessino and Fenochietoo 2010). Other inequality measures include the Jain's index and the Shannon entropy.

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An important observation is that the ultimate goal of a fairness measure is to *shape an allocation distribution* in a desirable way. As such, the utility allocation problem can also be interpreted as a stochastic programming (SP) problem in which one seeks to find allocations that shape a distribution of outcomes (in SP the outcome distribution is shaped by using a risk measure) (Dowling et al. 2016, Hu and Mehrotra 2012). As in the case of fairness measures, axioms have been proposed in the SP literature to study the selection of suitable risk measures (Artzner et al. 1999). These properties have been recently exploited to identify compromise solutions that maximize the collective satisfaction of multiple stakeholders (Dowling et al. 2016).

Fundamental Axioms of a Fair Allocation

In seminal work, Nash (1950) proposed a set of axioms (Axioms 1-4 listed below) that a solution to a two-stakeholder utility allocation problem should satisfy. Nash proved that there exists a unique solution satisfying such axioms, that is now known as the Nash solution. These results have been extended to n-stakeholder settings in Roth (1979a). The fundamental axioms that a fair allocation must satisfy are:

Axiom 1. *Pareto Optimality:* Ensures there is no wastage of utilities/resources.
Axiom 2. *Symmetry (Anonymity):* Ensures there is no discrimination among stakeholders.
Axiom 3. *Affine Invariance (AI):* Ensures that the solution is *scale-invariant*.
Axiom 4. *Independence of Irrelevant Alternatives (IIA):* Ensures that the choice of a utility allocation over another is not affected by irrelevant utilities in the set.
Axiom 5. *Restricted Monotonicity:* Implies that, if the feasible utility set is expanded, then the allocation under the expanded set should dominate that in the original set. This condition can be difficult to satisfy as it can prevent trading-off even a small portion of an allocation in exchange for a major allocation gain for another stakeholder.

The axiomatic properties of different utility allocation schemes are summarized in Table 1. We note that no scheme can satisfy Axioms 1-5 simultaneously (Roth 1979b). In particular, of all the schemes analyzed, only the two-stakeholder Kalai-Smorodinsky scheme satisfies restricted monotonicity. We also see that the Nash, Shannon entropy, and generalized entropy schemes satisfy the most axioms (four out of five) but Nash satisfies AI (while entropy approaches only satisfy weak AI). We also note that the Nash scheme does not assume a fixed total utility (and therefore it is a more flexible approach than entropy approaches). Notably, the social welfare scheme only satisfies three out of five axioms (and only satisfies weak AI).

Table 1. Summary of axiomatic properties

Scheme	Pareto	Symmetry	AI	IIA	Monotonicity
Social Welfare	✓	✗	✗	✓	✗
Nash	✓	✓	✓	✓	✗
Kalai-Smorodinsky with $n = 2$	✓	✓	✓	✗	✓
Max-Min	✗	✗	✗	✓	✗
α-Fair with $\alpha \in (2, \infty)$	✓	✓	✗	✓	✗
Entropy	✓	✓	✗	✓	✗
Superquantile with $\alpha \in (0, 1)$	✗	✗	✗	✓	✗
Gen. Entropy with $\beta \in [2, \infty)$	✓	✓	✗	✓	✗

Conclusion

We presented an axiomatic analysis of various utility allocation schemes and derived fundamental connections between them. Such an analysis can guide decision-makers in selecting a suitable measure to allocate utilities among multiple stakeholders.

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