A Decomposition Algorithm for Simultaneous Scheduling and Control of CSP Systems

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Abstract

We present a decomposition algorithm to perform simultaneous scheduling and control decisions in concentrated solar power (CSP) systems. Our algorithm is motivated by the need to determine optimal market participation strategies at multiple timescales. The decomposition scheme uses physical insights to create surrogate linear models that are embedded within a mixed-integer linear scheduling layer to perform discrete (operational mode) decisions. The schedules are then validated for physical feasibility in a dynamic optimization layer that uses a continuous full-resolution dynamic CSP model. The dynamic optimization layer updates the physical variables of the surrogate models to refine schedules. We demonstrate that performing this procedure recursively provides high-quality solutions of the simultaneous scheduling and control problem. We exploit these capabilities to analyze different market participation strategies and to explore the influence of key design variables on revenue. Our results also indicate that using scheduling algorithms that neglect detailed dynamics significantly decreases market revenues.

Keywords: mixed integer programming, dynamic optimization, electricity markets, solar energy, thermal energy storage.

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Introduction

Electrical power infrastructures are being restructured to enable the adoption of intermittent solar and wind resources. Despite their environmental benefits, renewable sources introduce additional uncertainty over different timescales (seconds to seasonal). As a result, technologies that offer a broader spectrum of dynamic flexibility are becoming increasingly attractive. In particular, embedding energy storage technologies within generation systems such as concentrated solar power systems (CSP) is critical to ensuring security and reliability of power supply. Power grids use market-based mechanisms to coordinate electricity generation and consumption.\textsuperscript{8,13,42} Operating CSP systems under such dynamic market environments is complicated because price dynamics are often out-of-phase with solar irradiance (which is the main energy source in such technologies). In California, for instance, average prices reach their minimum around noon and double to their maximum shortly after sunset.\textsuperscript{15}

Recent literature considers direct participation of CSP systems in wholesale electricity markets.\textsuperscript{11,19,24,25,37} Market revenues for CSP systems are estimated by solving a mixed integer linear programming (MILP) problems that incorporate market rules, CSP physics, and historical market prices and weather conditions. Integer variables are used to model transitions between different operating modes (e.g., generating mode, warm, off, antifreeze, and so on).\textsuperscript{19} To enable computational tractability, linear steady-state models that track only energy flows and holdups are often considered.\textsuperscript{25} For example, a CSP market dispatch system based on MILPs was recently added to System Advisor Model (SAM), a ubiquitous software tool for techno-economic analysis of solar power systems.\textsuperscript{10,41} Market participation of CSP systems, however, requires capturing the impact of weather and market price dynamics at high temporal resolution\textsuperscript{24} in order to properly assess dynamic physical flexibility. In particular, dynamic models are needed to identify market opportunities at fast timescales (seconds to minutes), which are becoming increasingly profitable.\textsuperscript{13} The controls community has used de-
tailed dynamical models to capture nonlinear effects such as non-isothermal mixing and Rankine cycle efficiency.\textsuperscript{6,18,31,32} Combining scheduling and control (dynamic optimization) models results in non-convex mixed integer nonlinear programs (MINLPs) that cannot be solved by off-the-shelf solvers. Vasallo and Bravo\textsuperscript{38,39} propose a two-model approach to address this issue; here, a MILP is solved to determine the optimal generation schedule which is then simulated with a high-fidelity dynamic model implemented in SAM.\textsuperscript{10} The high-fidelity simulation results are then used to tune the optimization model. In this work, we propose an alternative framework where the high-fidelity simulation is replaced with a nonlinear dynamic optimization problem within the decomposition framework. We use physical insights to create a surrogate model for the scheduling layer that is progressively refined by the dynamic optimization layer. We use this framework to address the following research questions:

1. What is the economic impact of neglecting dynamic physical flexibility in scheduling decisions?

2. Which market timescales and products offer the most revenue potential for CSP systems?

3. How are market revenues affected by design decisions such as solar collector and storage tank sizes?

The remainder of this paper is organized as follows. Section 2 reviews electricity markets and summarizes mathematical models for market participation. Section 3 describes a high-fidelity dynamic model for parabolic trough CSP systems. The proposed decomposition algorithm is presented in Section 4. Section 5 gives computational results and discussion. Conclusions and future work are discussed in Section 6. Nomenclature is defined in the Appendix A and detailed mathematical models are given in Appendix B.
Electricity Market Participation Mechanisms

Electricity markets involve sophisticated hierarchical decision-making processes that coordinate generation and consumption in the power grid at multiple timescales. In North America, resources participate in markets by transacting two categories of products: electrical energy (measured in MWh) and ancillary service capacity (measured in MW). In California, energy is transacted at three timescales: in 1-hour intervals in the Day-Ahead Market (DAM, also known as the Integrated Forward Market), in 15-minute intervals in the Fifteen Minute Market (FMM), and in 5-minute intervals in the Real Time Dispatch Process (RTD). Together, the FMM and RTD are known as the Real-Time Market (RTM). Figure 1 shows historical energy prices for seven days near Daggett, CA. Notably, prices are highest and most volatile in the evening. The lowest prices occur around midday, likely due to large amounts of distributed photovoltaic (PV) panels in CA. Significant price volatility can be observed in the RTM in the mornings, which corresponds to PV resources ramping up. Table 3 gives descriptive statistics for the entire year. Although the DAM prices are 1.4 to 1.5 $/MWh higher on average, the RTM prices are three to six times more volatile. These cyclic and volatile prices create significant incentives for energy storage. For CSP systems, up to 400,000 $/year of additional revenue are available by shifting 10 MW of generation from the average real-time energy price ($ 30 /MWh) to the 1% most extreme prices (97 to 1621 $/MWh). Computational results show how CSP systems can miss significant revenue opportunities without RTM participation.

Besides conducting direct energy transactions, power generation technologies can also participate in electricity markets by providing ancillary services such as frequency regulation, spinning reserves, and non-spinning reserves. Ancillary services are a contingency product where resources are paid...
for providing a power capacity (prices are in $/MW). Reserve capacity is used as contingency against unplanned events that cannot be mitigated alone by RTM energy transactions, such as wind and solar supply variations, generator failures, and large errors in load forecasts. For example, a generator providing spinning reserves for CAISO is contractually obligated to fully respond within 10 minutes of dispatch and provide 30 to 60 minutes of power at the awarded capacity. A resource providing reserves is paid both for the awarded capacity and then for energy provided during dispatch. Furthermore, reserve dispatches are rare and occur about fifty times per year. As discussed in Section , spinning reserves are especially attractive for CSP systems where the steam turbines are typically idle. Moreover, with sufficient energy storage, a CSP system can operate in spinning mode (turbine is synchronized with the grid but not generating electricity) which both provides supplemental revenue for the generator and helps ensure grid reliability.

*Frequency regulation* (FR) is another important revenue opportunity for CSP systems. As the name implies, the FR ancillary service is used to help stabilize the AC power frequency of the grid (e.g., 60 Hz in North America, 50 Hz in Europe). FR capacity is bid to the market as a *flexibility band*. Every 2 to 15 seconds, the grid sends each participating resource a new power set-point within the awarded flexibility band. Resources are paid for both the size of the band, known as capacity payments, and the variability of the set-point signal, known as mileage payments. A CSP system providing 10 MW of FR capacity for all hours of 2015 in the California market would have received $500,000 in capacity payments alone. In order to provide FR capacity, a resource must have sufficient fast dynamic flexibility on the order of seconds to minutes, which is why FR is the most expensive ancillary service. Dowling and Zavala recently found that FR dispatch signals are predominantly composed of fast harmonics (seconds to hours) which are naturally attenuated by slow process dynamics. Thus, CSP systems should be able to provide frequency regulation by exploiting dynamic flexibility from thermal energy storage. Coal-fired generators, which are also based on Rankine cycles, have been providing FR capacity for decades.
A key challenge for market participants is to determine how to best allocate their generation capacity among market products and timescales. This is especially challenging given complex market rules, physical limitations of the technology, and multiscale correlations embedded in coincidental market prices and weather. As such, most existing market dispatch frameworks and techno-economic assessments for CSP systems, such as, focus solely on the day-ahead market or only energy transactions or both. Dowling et.al. recently proposed an optimization framework to identify optimal participation strategies in multiscale and multiproduct electricity markets. The modeling abstraction used in is general purpose and can be coupled to detailed physical models of different technologies. We now proceed by describing the basic components of the market participation model. For more details, the reader is referred to.

We define the sets of time intervals \( \mathcal{T}_\ell := \{1, \ldots, N_\ell\} \) where \( \ell \) indicates the market layer and \( \ell \in \mathcal{L} := \{3, 2, 1, 0\} \) and \( \Delta t_\ell \) denotes the length of the time interval in layer \( \ell \in \mathcal{L} \) (units of hours). For markets in CAISO, \( N_1 = 24 (\Delta t_1 = 1 \text{ hour}) \), \( N_2 = 4 (\Delta t_2 = 15 \text{ min.}) \), and \( N_3 = 3 (\Delta t_3 = 5 \text{ min.}) \). The lexicographic time set:

\[
\mathcal{T}^* := \mathcal{T}_{3} \times \cdots \times \mathcal{T}_2 \times \mathcal{T}_1 \times \mathcal{T}_0 \\
= \{(1, 1, 1, 1), (2, 1, 1, 1), \ldots, (N_3, 1, 1, 1), (1, 2, 1, 1), \ldots, (N_3, N_2, 1, 1), \ldots, (N_3, N_2, N_1, N_0)\} \tag{0.1}
\]

captures the hierarchical nature of the time discretization. Lexicographic time sets for individual layers are similarly defined: \( \mathcal{T}_3^* := \mathcal{T}^*, \mathcal{T}_2^* := \mathcal{T}_2 \times \mathcal{T}_1 \times \mathcal{T}_0, \mathcal{T}_1^* := \mathcal{T}_1 \times \mathcal{T}_0, \) and \( \mathcal{T}_0^* = \mathcal{T}_0 \). Thus, an instance in time \( t \) is defined by the tuple \((i_3, i_2, i_1, i_0)\) such that \((i_3, i_2, i_1, i_0) \in \mathcal{T}_3\), \((i_2, i_1, i_0) \in \mathcal{T}_2\), and so on. We use \( \bar{E}_{\ell,t} \) and \( E_{\ell,t} \) for \( t \in \mathcal{T}_\ell \) to represent the energy sales and purchases, respectively, for market layer \( \ell \). Similarly, we use \( A_{a,\ell,t} \) to represent ancillary service capacity sales, where \( a \in \mathcal{A} := \{s, n, r^+, r^-\} \) (for spinning reserves, non-spinning reserves, regulation up, and regulation down).
The total market revenues, denoted by $R$, are calculated as follows:

$$R = \Lambda \sum_{\ell \in L} \sum_{t \in T} \left[ \Delta_t \pi^E_{\ell,t} (\bar{E}_{\ell,t} - (1 + \epsilon)\bar{E}_{\ell,t}) + \sum_{a \in A} \pi^A_{a,\ell,t} A_{a,\ell,t} \right]$$ \hspace{1cm} (0.2)

where $\pi^E$ and $\pi^A$ are the time-vary prices for energy and ancillary service capacity, $\epsilon$ is a small number (e.g., $10^{-6}$), and $\Lambda$ is the nameplate capacity of the generator. We use $E_t$ to represent the net energy flow out of the energy system which is defined at the fastest timescale as follows:

$$E_t = \hat{E}_t + \sum_{\ell \in L} (\bar{E}_{\ell,t} - E_{\ell,t}), \quad t \in T^*$$ \hspace{1cm} (0.3)

where $\hat{E}_t$ represents the flow of energy to onsite demands (such as pumps and other auxiliary loads).

Under this basic market participation abstraction, the detailed energy systems model only needs to interface with variables $E_t$ and $\hat{E}_t$. All energy and ancillary service capacity variables are scaled by the nameplate capacity, denoted by $\Lambda$, such that $0 \leq E_t, \hat{E}_t, \bar{E}_{\ell,t}, E_{\ell,t}, A_{a,\ell,t} \leq 1$. A 3% per minute ramp rate restriction is imposed on $E_t$ in the market model (see\textsuperscript{13}), which is consistent with the ramping limits for other Rankine cycles.\textsuperscript{4}

### Mathematical Model and Optimization Formulation

CSP systems transform thermal energy collected from the sun into electricity. As shown in Figure 2, CSP systems are organized into two interconnected loops. In loop 1, an advanced organic heat transfer fluid (HTF) such as Dowtherm A\textsuperscript{1} is heated in the parabolic trough solar collectors and stored in the hot storage tank. When the CSP system generates electricity, the hot HTF is pumped into the warm storage tank via heat exchangers. Heat is transferred from loop 1 into loop 2, which is a standard regenerative Rankine cycle. This configuration allows for energy collection and conversion to be partially decoupled such that electricity generation can occur during times with low solar irradiance. This configuration, however, introduces control challenges for CSP systems. In particular, it is

\textsuperscript{1}http://www.dow.com/heattrans/csp/index.htm
necessary to coordinate mass and energy holdups in the storage system while ensuring operational
restrictions of the individual loops.\textsuperscript{44}

[Figure 2 about here.]

Strategic market participation for CSP systems can be formulated as the following mixed integer
dynamic optimization problem:

\begin{align*}
\max_{z(t), x(t), u(t), y(t)} & \int_0^{T_f} \pi^t u(t) \, dt \\
\text{s.t.} & \quad Au(t) + By(t) \leq 0 \\
& \quad \frac{dz}{dt} = f(z(t), x(t), u(t)) \\
& \quad g(z(t), x(t), u(t)) = 0 \\
& \quad z \leq z(t) \leq \bar{z} \\
& \quad x \leq x(t) \leq \bar{x} \\
& \quad u \leq u(t) \leq \bar{u}
\end{align*}

where $z(t) \in \mathbb{R}^{n_z}$ are differential states, $x(t) \in \mathbb{R}^{n_x}$ are algebraic states, $u(t) \in \mathbb{R}^{n_u}$ are continuous
control variables, and $y(t) \in \{0, 1\}^{n_y}$ are discrete control variables. The most important variables
in the CSP mathematical model are shown in Figure 2. Differential states track the time-evolution
of mass ($M_{1,t}, M_{2,t}$) and temperature ($T_{1,t}, T_{2,t}$) in thermal storages tanks 1 and 2. Algebraic states
include the gross power output of the turbines ($W_t$) as well as HTF temperature leaving the solar
collector field ($T_{s,t}$) and Rankine cycle ($T_{r,t}$); these are calculated from algebraic performance correla-
tions.\textsuperscript{27} Continuous control variables include HTF mass flow rates through the solar collectors ($\dot{m}_{s,t}$)
and Rankine cycle ($\dot{m}_{r,s}$) as well as capacity allocation to market products ($\bar{E}_{\ell,t}, A_{a,\ell,t}$, etc). Finally,
discrete control variables $y_{e,t}$ and $y_{sp,t}$ describe if the CSP system is in \textit{ON}, \textit{SPIN}, or \textit{OFF} modes.
The objective function of (P1) is often to maximize revenue, as given in (0.2). The market rules (described in 13) and operation restrictions are modeled linear constraints (P2b). Mass and energy balances for the storage tanks give rise to the nonlinear differential equations (P2c). Similarly, nonlinear performance correlations for the solar collectors and Rankine cycle comprise the algebraic constraints (P2d). Finally, the differential and algebraic states are subject to path constraints (bounds). The complete CSP physics model is reported in Appendix B.

Problem (P1) is discretized with a 5-minute timestep, which corresponds with the fastest market layer, resulting in a complex nonconvex Mixed Integer Nonlinear Problem (MINLP). For example, with a scheduling horizon of only one day, (P1) contains 8250 continuous variables, 72 binary variables, 9036 linear constraints and 2916 nonlinear constraints. About 20% of the continuous variables and 30% of the linear constraints arise from the market participation model. Despite the fact that this is an MINLP of moderate size, the off-the-shelf global solver SCIP 4.0 often did not advance beyond the initial point and occasionally failed to compute valid bounds. Bonmin was unable to solve (P1) for planning horizons longer than three days (see Computational Results section).

Decomposition Algorithm

In order for (P1) to be used for daily market participation decisions, it needs to be solved in less than a few hours for a multi-day planning horizons. To meet this computational budget, we propose a decomposition algorithm, which we refer to as (AR1):
Data: Market prices, solar irradiance, and other model parameters

for $k = 1$ to $N$ do

if $k = 1$ then

Load data and assemble (P2.1)

Solve (P2.1) and store results in $v_{2,k}^*$

else

Assemble (P2.2) using results $v_{3,k-1}^*$

Solve (P2.2) and store results in $v_{3,k}^*$

end

Fix discrete variables at values in $v_{2,k}^*$

Simulate CSP system using $v_{2,k}^*$ and initialize (P3)

Solve (P3) and store results in $v_{3,k}^*$

Merge $v_{2,k}^*$ and $v_{3,k}^*$ to form $v_{1,k}^*$, a feasible solution for (P1)

end

Algorithm 1: Decomposition algorithm (AR1).

The key insight in (AR1) is to decompose the full problem (P1), and iterate between scheduling optimization problems (P2) and nonlinear dynamic optimization problems (P3). Two formulations for the scheduling problem as considered: formulation (P2.1) is used for initialization and formulation (P2.2) is used for subsequent iterations. Both formulations are given in Appendix B. The solution of (P2), which is stored in $v_{2,k}^*$, contains the time trajectory for the discrete operating mode decisions $y_{e,t}$ and $y_{sp,t}$ (these have a time resolution of one hour). These variables are fixed for in the dynamic optimization problem (P3), which contains the full-resolution CSP physics with 5-minute time discretization (this corresponds to the faster CAISO market layer). For each subsequent iteration of (AR1), the surrogate model used in (P2) is updated based on the the solution of (P3) from the previous iteration (which is contained in $v_{3,k-1}^*$). The proposed decomposition approach has the advantage that (P2) and (P3) can be solved with off-the-shelf solvers. Moreover, the surrogate physical
models used in (P2) are constructed to be consistent with (P3), which leads to favorable numerical results. The remainder of this section details the formulations for (P2) and (P3).

### Scheduling Problem

The goal of problem (P2) is to compute schedule operational modes. To accomplish this, the full-resolution CSP physics model is simplified into linear constraints, \( \hat{f}(\cdot) \) and \( \hat{g}(\cdot) \), to create a surrogate model:

\[
\begin{align*}
\max_{\hat{z}(t), x(t), u(t), y(t)} & \quad \int_0^{T_f} \pi^T u(t) \, dt \\
\text{s.t.} \quad & Au(t) + By(t) \leq 0 \quad \text{(P2b)} \\
& \frac{d\hat{z}}{dt} = \hat{f}(\hat{z}(t), x(t), u(t)) \quad \text{(P2c)} \\
& \hat{g}(\hat{z}(t), x(t), u(t)) = 0 \quad \text{(P2d)} \\
& \hat{z} \leq \hat{z}(t) \leq \bar{\hat{z}} \quad \text{(P2e)} \\
& \underline{x} \leq x(t) \leq \bar{x} \quad \text{(P2f)} \\
& \underline{u} \leq u(t) \leq \bar{u} \quad \text{(P2g)}
\end{align*}
\]

We propose a tailored linearization from \( f(\cdot) \) and \( g(\cdot) \). In the full-resolution model, storage tank variables are differential states, i.e., \( z(t) = [M_1, M_2, T_1, T_2]^T \), which introduces bilinear terms in the energy balances:

\[
E_{i,t} = C_p M_{i,t} T_{i,t}. \quad \text{(0.4)}
\]

In contrast, energy holdups in the storage tanks, \( E_1 \) and \( E_2 \), replace temperatures as state variable in simplified physical model, i.e., \( \hat{z}(t) = [M_1, M_2, E_1, E_2]^T \). As consequence of this change of basis, mass and energy balances become linear. This is a key distinction from MINLP algorithms such as Outer Approximation,\(^{17}\) which do not rely on physical insights to construct linearization. Moreover,
the proposed algorithm updates the linearizations \( \hat{f}(\cdot) \) and \( \hat{g}(\cdot) \) each iteration and does not, at present, use information from previous iterations (besides the most recent). In contrast, Outer Approximation (and similar algorithms) accumulates information from all iterations as cutting planes in the master problem.

We observe that temperature dynamics are slow with sufficiently large storage tanks, and thus, (0.4) can be approximated as follows:

\[
M_{i,t} (\overline{T}_{i,t} - \delta_T) \leq E_{i,t} C_p \frac{1}{\kappa_{kJ}} M_{i,t} \left( \overline{T}_{i,t}^\text{avg} + \delta_T \right), \quad i \in \{1, 2\},
\]

where \( \overline{T}_{i,t} \) which represents the temperature in tank \( i \) at time \( t \) from the solution of (P3) at iteration \( k - 1 \). We will use the notation \( \overline{x} \) to denote fixed values for variable \( x \) obtained from (P3) at iteration \( k - 1 \). Linear equations (0.5) couple mass and energy holdup state variables and ensure the new solution of (P2) is in a neighborhood near the previous solution of (P3) (i.e., where the simplified model is still valid). \( \delta_T \) is a tunable trust-region parameter that balances the accuracy of the simplified model and the exploration space allowed.\(^9\)

The proposed approach has two key distinctions from other MILP models in literature:

1. Mass flows and holdups are considered in addition to energy flows and energy holdups. Other simplified models in the literature only consider energy holdups.

2. A 5-minute timestep is used to match the fastest market layer, whereas most simplified models from literature use a 1-hour timestep.

These distinctions ensure that the proposed surrogate model is consistent with the full-resolution model. In particular, the surrogate model is constructed such that any feasible solution of (P3) must be feasible solution to the next instance of (P2). In practice, solutions of (P2) often give good starting points for (P3), although we cannot guarantee a feasible solution for (P3) for every set of discrete variables generated by (P2).
Dynamic Optimization Problem

The goal of the dynamic optimization problem (P3) is to find the optimal market participation and dispatch decisions for mass and energy flows for a fixed operating mode schedule obtained with (P2). This is formulated as follows,

\[
\max_{z(t), x(t), u(t)} \int_0^{T_f} \pi^T u(t) \, dt \quad \text{(P3a)}
\]

s.t.

\[
A u(t) + B y(t) \leq 0 \quad \text{(P3b)}
\]

\[
\frac{dz}{dt} = f(z(t), x(t), u(t)) \quad \text{(P3c)}
\]

\[
g(z(t), x(t), u(t)) = 0 \quad \text{(P3d)}
\]

\[
z \leq z(t) \leq \bar{z} \quad \text{(P3e)}
\]

\[
x \leq x(t) \leq \bar{x} \quad \text{(P3f)}
\]

\[
u \leq u(t) \leq \bar{u} \quad \text{(P3g)}
\]

where \( y(t) \) is fixed. Problem (P3) is discretized with a 5-minute timestep resulting in a nonconvex nonlinear program (NLP).

Results and Discussion

In this section, we present computational results and use the proposed decomposition algorithm quantify the revenue potential of CSP systems from different market layers and products. We also use the algorithm to explore effect of design variables on revenue potential.

[Figure 3 about here.]

[Figure 4 about here.]

[Table 2 about here.]
Computational Performance

Algorithm (AR1) was used to optimize the market participation strategy for a parabolic trough system interacting in the California market. Problem (P2) was solved with Gurobi 7.0.2\textsuperscript{21} with a MIP gap of 0.5% and (P3) was solved with Ipopt 3.12.4\textsuperscript{40} using the MA27 linear algebra routines.\textsuperscript{20} Historical market prices and weather data were obtained from oasis.caiso.com and, respectively, for January 1 - 7, 2015. We note that the revenue estimates reported herein are upper bounds, as we assume exact forecasts. We will consider uncertainty as future work and note that Sioshansi and Denholm\textsuperscript{34} report obtaining over 90% of available DAM revenues with simple forecasting strategies. The CSP model is based on\textsuperscript{27} and comprises a 35 MW\textsubscript{e} net CSP generator. As a reference, the world’s largest CSP plant, Ivanpah Solar, is 10 times larger\textsuperscript{2}. For the seven day planning horizon under study, (P2.2) contained approximately 39,000 continuous variables, 500 binary variables, and 75,000 linear constraints. Problem (P3) contained approximately 64,000 continuous variables, 37,000 linear constraints, and 18,000 nonlinear constrains. The decomposition algorithm was implemented in the Julia programming language using the JuMP modeling environment.\textsuperscript{5,16}

The first case study explores evolution of the objective function (the total revenue) for 25 iterations of (AG1). Figures 3 and 4 present the revenue and computational times, respectively, with trust-region parameters $\delta_T = 10 \, ^\circ\mathrm{C}$ and $30 \, ^\circ\mathrm{C}$. Table 4 reports results for $\delta_T = 1 \, ^\circ\mathrm{C}$, $15 \, ^\circ\mathrm{C}$, $20 \, ^\circ\mathrm{C}$, $25 \, ^\circ\mathrm{C}$, and $100 \, ^\circ\mathrm{C}$ too. From these results we can draw the following conclusions:

- With $\delta_T = 10 \, ^\circ\mathrm{C}$ and $30 \, ^\circ\mathrm{C}$, the revenue obtained with the scheduling model (P2) is about half of the best revenue obtained. This implies that the original surrogate CSP model used in the scheduling model poorly captures the dynamic flexibility of the CSP system.

- The algorithm (AG1) progressively improves the revenue and eventually settles. In particular, the revenue of (P3) is improved by nearly 35% (relative to the first iteration). We also note

\textsuperscript{2}http://www.nrel.gov/csp/solarpaces/
that the gap in revenue between the scheduling (P2) and dynamic optimization (P3) models eventually dies out.

- The trust-region parameter $\delta_T$ balances the rate of convergence, computational times, and the best revenue (objective) value. From Figure 3, (AG1) converges in about 8 iterations with both $\delta_T = 10 \degree C$ and $\delta_T = 30 \degree C$. At the extreme $\delta_T = 1 \degree C$, (AR1) converged after only two iterations to a local solution that only captures 77% of the available revenue. Table 4 shows that, in general, as $\delta_T$ increases, the computational demands also increase and shift from (P2) to (P3).

- For a given iteration, the objective values for (P3) are better than those of (P2). This highlights that (P2) is not a relaxation of (P3) but instead a linearization (approximation). Moreover in early iterations the linear model in (P2) underpredicts revenue and missing opportunities only available from exploit dynamics and nonlinear effects.

We hypothesize the scheduling decisions are near optimal because the linear surrogate models $\hat{f}(\cdot)$ and $\hat{g}(\cdot)$ locally approximate the full-resolution dynamic model. This was tested by solving (P1), discretized with 5-minute timesteps, using the four MINLP algorithms available with Bonmin. The solution for (AR1) was used as an initial point for each instance solved with Bonmin. The results are shown in Table 5. We highlight that all of the solutions obtained with (AR1) were within 2% of best objective found with Bonmin, and for half of the instances, the difference was less than 0.5%. Algorithm (AR1) is 3 to 300 times faster than Bonmin. We note that Bonmin was unable to solve for planning horizons longer than 3 days, whereas (AR1) has been demonstrated with 7-day planning horizons (Figures 5 - 7). We hypothesize (AR1) is more computationally efficient because the linearization strategy exploits physical insights for the system, where Bonmin is a general purpose MINLP solver. Algorithm (AR1) is not guaranteed to provide an optimal solution but in practice it can be used to obtain feasible solutions of good quality. We will study the convergence properties
Market Participation

In a second case study, we used algorithm (AR1) to quantify the revenue opportunities from different market products and timescales. Figure 5 compares twelve different market participation strategies, organized into three groups: participation in both markets, only the day-ahead market, and only the real-time market. Four combinations of market products are considered for each group: a) energy only, b) energy and regulation, c) energy and spinning reserves, and d) energy, regulation and spinning reserves. The pattern in each bar denotes the total revenue from each timescale. Revenues are extrapolated to one year for a better sense of scale. We draw the following conclusions:

• Market participation in day-ahead markets only captures 60% (energy and ancillary services) to 70% (energy only) of the available revenue obtained from participation in both markets. In contrast, participating in the real-time market alone captures over 85% of the available revenues. This highlights the importance of fast flexibility and the need to consider dynamic models in scheduling decisions.

• Ancillary services offer substantial new revenue streams for CSP systems: up to 50% (DAM), 30% (RTM), and 29% (both) revenue improvements are obtained relative to energy-only market participation.

• For DAM participation, spinning reserves is the most valuable ancillary service, increasing revenues 30% relative to energy only participation. These results are in agreement with, who found a 17% increase in revenue from spinning reserves. However, when participating in the RTM or both markets, regulation is the most valuable ancillary service.
We now examine the operational policies obtained for full market participation (for all products and timescales) in more detail to elucidate how the CSP system exploits dynamic flexibility to maximize revenues. Figure 6a gives the optimal allocation of generation capacity into market products. Energy sales are shown in purple (with cross pattern) with the average generation level marked with the solid black line. Regulation up and down capacity are shown in red and blue, respectively. Recall that regulation is an ancillary service where the generator provides a flexibility band. This means that the actual energy production will be between the bottom of the blue region (reg. down) and the top of the red region (reg. up). Careful inspection reveals that the regulation down capacity (blue) overall with some of the energy sales (purple). Finally, spinning reserve capacity is shown in green.

Figure 6b shows the optimal control strategy, including the two primary energy flows, collection of solar heat and electricity generation, as well as energy levels in the hot and warm storage tanks. From Figure 6, we draw the following conclusions:

- Optimal energy sales often correspond to periods of high prices (see Figure 1), in particular during the morning and evening. Energy is sometimes sold during times of medium or low prices to help manage the mass and energy holdup in the storage tanks.

- Energy sales are always paired with selling regulation capacity. When the regulation down capacity price is higher than the up capacity price, it is optimal to set nominal generation between 90% and 100% and sell a 20 to 30% regulation down capacity. Similarly, when regulation up is more expensive, it is optimal to set nominal generation between 60% and 70% and sell the remaining capacity as regulation up. Likewise, spinning reserve prices are sometimes higher than and displace regulation up capacity sales.

- Spinning reserves allow CSP systems to capture revenue when they would otherwise be off. This occurs only a few times during the 7-day horizon shown in Figure 6a. This is because,
for these results $\xi = 0.05$, which means that operating in spinning mode incurs an energy penalty of 5% of the nameplate capacity. When spinning mode losses are not considered ($\xi = 0$, see\textsuperscript{12}), spinning reserves are almost always sold instead transitioning into off mode. Detailed modeling to determine the value for $\xi$ is beyond the scope of this work.

- The hot tank storage is almost completely emptied of useful energy each night, suggesting there is little coupling between days. This aligns with the observations of\textsuperscript{14} where the authors found that most market volatility and hence revenue opportunities are intra-day.

[Figure 7 about here.]

**Effects of Design**

The third and final case study examines the impact of two design decisions, solar field size and storage tank size, on market revenues. In this study, all market timescales are considered and we vary the type of products provided. The nominal configuration was a solar multiple ($\Theta$) of 1.5 with 8 hours of thermal storage (output at full nameplate capacity), which was used for the previous results. Figure 7 shows sensitivity analysis results around this nominal design for these market participation strategies. Absolute revenues (extrapolated to one full year) are shown for the energy only case. The relative increases in revenue are reported for all other product combinations. We make the following observations:

- For some trials in the solar multiple sensitivity analysis, such as $\Theta = 1.75$ (E + R + SR), $\Theta = 2.0$ (E + SR, E + R), $\Theta = 2.5$ (E, E + SR), revenues are less than overall trends. It is likely that for these points (AR1) found solutions that are not globally optimal.

- With energy-only participation, revenues flatten out for solar multiples above 1.5. With simultaneous energy and ancillary service sales, however, revenues do not flatten out and continue to increase as the solar multiple increases.
Over the solar field sizes considered, full market participation increased revenues 20% to 37% over energy only participation. Similarly, ancillary services increased revenues 14% to 34% over the range of storage sizes considered.

Revenues almost doubled over the range of storage sizes considered: from 4.4 M$/yr with 2 hours of storage to 8.2 M$/yr with 12 hours of storage (energy only). Gains from varying the doubling the solar field size were more modest.

Conclusions and Future Work

We present a computationally efficient decomposition algorithm to perform simultaneous scheduling and control decisions in CSP systems. Physical insights inform an embedded, tunable surrogate linear model that couples mixed-integer linear scheduling layer and a dynamic optimization layer. With this algorithm, we find dramatic revenue opportunities for CSP systems participating in multiscale electricity markets. We show how up to 40% of revenue opportunities are only accessible through the real-time market, yet most literature neglects these fast timescales. We also find that selling ancillary service capacity can increase market revenues up to 50%. As such, advanced control strategies and market participation offers CSP technologies means to improve their value proposition relative to other renewable technologies without dramatic increases in capital costs.

As future work, we plan to consider design and market participation co-optimization similar to\textsuperscript{23,26} as well as probabilistic weather and price forecasts similar to\textsuperscript{11,28–30,43} Yet modeling multiscale effects (minutes to decades) and uncertainty often result in extremely large optimization problems. To address the computation challenges, we propose exploiting problem structure and using massively parallel optimization solvers such as\textsuperscript{7,22} for the subproblems (P2) and (P3). We also plan to apply this decomposition algorithm to other chemical processes and thermal energy storage systems.
Acknowledgments

The authors thank The Dow Chemical Company for their generous financial support. This contribution was identified by Wesley Cole (National Renewable Energy Laboratory, Golden, CO, USA) as the Best Presentation in the session “Sustainable Electricity: Generation and Storage” of the 2016 AIChE Annual Meeting in San Francisco. Finally, we would like to thank two anonymous reviewers for their constructive comments and suggested improvements.

Appendices

A Nomenclature

Table 1: Parameters Definitions and Constants

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value/Range</th>
<th>Units</th>
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<tbody>
<tr>
<td>$\alpha_0 - \beta_1$</td>
<td>Coefficients in field heat loss correlation</td>
<td>Table 6</td>
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</tr>
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<td>$\gamma_0 - \gamma_8$</td>
<td>Coefficients in Rankine cycle work correlation</td>
<td>Table 7</td>
<td></td>
</tr>
<tr>
<td>$\delta_T$</td>
<td>Tunable trust-region parameter</td>
<td>varies</td>
<td>[°C]</td>
</tr>
<tr>
<td>$\delta_m$</td>
<td>Tunable trust-region parameter</td>
<td>1.5</td>
<td>[-]</td>
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<td>$\Delta t_1$</td>
<td>Timestep for market layer 1</td>
<td>1</td>
<td>hour</td>
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<tr>
<td>$\Delta t_2$</td>
<td>Timestep for market layer 2</td>
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<td>min.</td>
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<tr>
<td>$\Delta t_3$</td>
<td>Timestep for market layer 3</td>
<td>5</td>
<td>min.</td>
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<td>$\Delta \bar{T}_{\text{SPIN}}$</td>
<td>Maximum temperature difference during SPIN mode</td>
<td>50</td>
<td>°C</td>
</tr>
<tr>
<td>$\Delta T_{\text{SPIN}}$</td>
<td>Minimum temperature difference during SPIN mode</td>
<td>200</td>
<td>°C</td>
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<tr>
<td>$\Delta \bar{T}_r$</td>
<td>Maximum temperature difference across Rankine cycle</td>
<td>400</td>
<td>°C</td>
</tr>
<tr>
<td>$\zeta^A_B$</td>
<td>Conversion factor from units “A” to units “B”</td>
<td>varies</td>
<td>B / A</td>
</tr>
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</table>
Table 1: Parameters Definitions and Constants

<table>
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<th>Units</th>
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<tr>
<td>$\epsilon$</td>
<td>Small number used in (0.2)</td>
<td>$10^{-6}$</td>
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<td>$\epsilon^2$</td>
<td>Small number for smoothed max operator, (B.30)</td>
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<tr>
<td>$\eta_{\text{solar}}$</td>
<td>Solar collector lumped heat-to-heat efficiency</td>
<td>71.33%</td>
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<tr>
<td>$\eta_{\text{pump}}$</td>
<td>Pump efficiency for solar field</td>
<td>60%</td>
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<td>$\eta_r$</td>
<td>Rankine cycle efficiency, used only for (P2.1)</td>
<td>0.35</td>
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<tr>
<td>$\theta_t$</td>
<td>Solar irradiance during time $(t - 1, t]$</td>
<td>0 - 1100</td>
<td>W / m²</td>
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<tr>
<td>$\theta_{\text{min}}$</td>
<td>Cut-off solar irradiance</td>
<td>50.0</td>
<td>W / m²</td>
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<td>$\Theta$</td>
<td>Solar area multiplier (i.e., solar multiple)</td>
<td>1.0 - 3.0</td>
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<td>$\kappa$</td>
<td>Pressure drop coefficient for solar field</td>
<td>13.18</td>
<td>s² / kg²</td>
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<tr>
<td>$\lambda$</td>
<td>Minimum operating capacity</td>
<td>0.3</td>
<td>[MW / MW]</td>
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<td>$\Lambda$</td>
<td>Nameplate capacity for Rankine cycle</td>
<td>35.0</td>
<td>MW</td>
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<td>$\mu_x$</td>
<td>Lower bound to relax $x_t$</td>
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<td>MW</td>
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<tr>
<td>$\bar{\mu}_x$</td>
<td>Upper bound to relax $x_t$</td>
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<td>$\mu_{\text{SPIN}}$</td>
<td>Minimum value for $\dot{m}_{r,t}$ in spinning mode</td>
<td>1.0</td>
<td>kg / s</td>
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<tr>
<td>$\mu_{\text{start}}$</td>
<td>Maximum value for $\dot{m}_r$ immediately after start-up</td>
<td>150.0</td>
<td>kg / s</td>
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<tr>
<td>$\mu_{\text{stop}}$</td>
<td>Maximum value for $\dot{m}_r$ immediately before shutdown</td>
<td>150.0</td>
<td>kg / s</td>
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<tr>
<td>$\mu_{\text{ON}}$</td>
<td>Minimum value for $\dot{m}_{r,t}$ in ON mode. See²⁷</td>
<td>150.0</td>
<td>kg / s</td>
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<tr>
<td>$\mu_{\text{SPIN}}$</td>
<td>Minimum value for $\dot{m}_{r,t}$ in SPIN mode. See²⁷</td>
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<td>kg / s</td>
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<td>$\bar{\mu}$</td>
<td>Maximum value for $\dot{m}_{r,t}$. See²⁷</td>
<td>500.0</td>
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<td>$\bar{\mu}_z$</td>
<td>Big-M parameter used to relax $z$</td>
<td>400</td>
<td>°C</td>
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<td>Big-M parameter used to relax $z$</td>
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<td>°C</td>
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<td>$\mu^*$</td>
<td>Intermediate HTF flow rate through Rankine cycle</td>
<td>350</td>
<td>kg / s</td>
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Table 1: Parameters Definitions and Constants

<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value/Range</th>
<th>Units</th>
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<tbody>
<tr>
<td>$\xi$</td>
<td>Power required to maintain spinning reserves mode</td>
<td>0.05</td>
<td>[MW / MW]</td>
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<tr>
<td>$\pi_{a,\ell,t}^A$</td>
<td>Price for ancillary service $a$ in market $\ell$ at time $t$</td>
<td>varies</td>
<td>$$/ MW</td>
</tr>
<tr>
<td>$\pi_{\ell,t}^E$</td>
<td>Price for electricity in market $\ell$ at time $t$</td>
<td>varies</td>
<td>$$/ MWh</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of heat transfer fluid</td>
<td>814.46</td>
<td>kg / m$^3$</td>
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<tr>
<td>$\sigma_t$</td>
<td>Reference value for $\dot{m}_{r,t}$ during SPIN mode</td>
<td>varies</td>
<td>kg / s</td>
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<td>$\bar{\sigma}$</td>
<td>Maximum value for $\dot{m}_{r,t}$ during SPIN mode</td>
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<td>Coefficients in Rankine cycle temperature correlation</td>
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<td>$\tau_{ON}$</td>
<td>Minimum time on after transition</td>
<td>3</td>
<td>hours</td>
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<td>$\tau_{OFF}$</td>
<td>Minimum time off after transition</td>
<td>2</td>
<td>hours</td>
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<td>$\bar{\chi}_{OFF}$</td>
<td>Upper bound for slack variable $x$ during OFF mode</td>
<td>12.678</td>
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<td>$\chi_{OFF}$</td>
<td>Lower bound for slack variable $x$ during OFF mode</td>
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<td>$\bar{\chi}_t$</td>
<td>Upper bound for slack variable $x$, used in (P2.2)</td>
<td>varies</td>
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<tr>
<td>$\chi_t$</td>
<td>Lower bound for slack variable $x$, used in (P2.2)</td>
<td>varies</td>
<td>MW</td>
</tr>
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<td>$\chi_{0,t}$</td>
<td>Value for slack $x$, used in (P2.2)</td>
<td>varies</td>
<td>MW</td>
</tr>
<tr>
<td>$\omega_{mirror}$</td>
<td>Collector mirror width</td>
<td>5.0</td>
<td>m</td>
</tr>
<tr>
<td>$A_{solar}$</td>
<td>Reference solar collector area ($\Theta = 1$)</td>
<td>182,000</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat for heat transfer fluid</td>
<td>2.4</td>
<td>kJ / kg-K</td>
</tr>
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<td>$I_{ON}$</td>
<td>Number of hours steam cycle must initially be on</td>
<td>0</td>
<td>[-]</td>
</tr>
<tr>
<td>$I_{OFF}$</td>
<td>Number of hours steam cycle must initially be off</td>
<td>0</td>
<td>[-]</td>
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Table 1: Parameters Definitions and Constants

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<th>Description</th>
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<th>Units</th>
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<tbody>
<tr>
<td>$T_{amb}$</td>
<td>Ambient temperature</td>
<td>25.0</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{use}$</td>
<td>Threshold for useful energy in (B.31) and (B.53)</td>
<td>250.0</td>
<td>°C</td>
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<tr>
<td>$T_i$</td>
<td>Lower bound for temperature of tank $i$</td>
<td>See below</td>
<td>°C</td>
</tr>
<tr>
<td>$\bar{T}_i$</td>
<td>Upper bound for temperature of tank $i$</td>
<td>See below</td>
<td>°C</td>
</tr>
<tr>
<td>$\bar{Q}_{r,t}$</td>
<td>Upper bound for $Q_{r,t}$, heat flow into Rankine cycle</td>
<td>See below</td>
<td>[MW / MW]</td>
</tr>
<tr>
<td>$UA$</td>
<td>Lumped heat transfer coefficient for storage tanks</td>
<td>See (A.6)</td>
<td>kJ / C - s</td>
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Table 2: Variables Definitions and Bounds

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<th>Bounds</th>
<th>Units</th>
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<tbody>
<tr>
<td>$g_t$</td>
<td>Slack variable used in (B.79)</td>
<td></td>
<td>kg/s</td>
</tr>
<tr>
<td>$m_{s,t}$</td>
<td>HTF flowrate through solar collectors during time $(t - 1, t]$</td>
<td>[0, 1000]</td>
<td>kg/s</td>
</tr>
<tr>
<td>$\dot{m}_{r,t}$</td>
<td>HTF flowrate through steam cycle during time $(t - 1, t]$</td>
<td>[0, 500]</td>
<td>kg/s</td>
</tr>
<tr>
<td>$M_{1,t}$</td>
<td>Total HTF mass in tank 1 at time $t$</td>
<td>[100, $\infty$]</td>
<td>tonnes</td>
</tr>
<tr>
<td>$M_{2,t}$</td>
<td>Total HTF mass in tank 2 at time $t$</td>
<td>[100, $\infty$]</td>
<td>tonnes</td>
</tr>
<tr>
<td>$\Delta P_{s,t}$</td>
<td>Pressure drop in solar field during time $(t - 1, t]$</td>
<td></td>
<td>bar</td>
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<tr>
<td>$P_{c,t}$</td>
<td>Condenser pressure during time $(t - 1, t]$</td>
<td>[0.08, 0.2]</td>
<td>bar</td>
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<tr>
<td>$T_{1,t}$</td>
<td>Temperature in tank 1 at time $t$</td>
<td>[150, 400]</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{2,t}$</td>
<td>Temperature in tank 2 at time $t$</td>
<td>[250, 400]</td>
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<tr>
<td>$T_{avg,i}$</td>
<td>Average temperature in tank $i$ during time $(t - 1, t]$</td>
<td>See above</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{s,t}$</td>
<td>Average exit temperature for solar field during time $(t - 1, t]$</td>
<td>[100, 400]</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{r,t}$</td>
<td>Average exit temperature for Rankine cycle during time $(t - 1, t]$</td>
<td>[0, 400]</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{r,t}^\dagger$</td>
<td>Typical exit Rankine cycle temperature</td>
<td>[100, 400]</td>
<td>°C</td>
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Table 2: Variables Definitions and Bounds

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<th>Units</th>
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<tbody>
<tr>
<td>$Q_{s,t}$</td>
<td>Heat collected in solar field during time $(t - 1, t]$</td>
<td>$[0, \infty]$</td>
<td>[MW / MW]</td>
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<tr>
<td>$Q^{\text{loss}}_{i,t}$</td>
<td>Average heat lost in tank $i$ during time $(t - 1, t]$</td>
<td>$[0, \infty]$</td>
<td>MW</td>
</tr>
<tr>
<td>$Q^{\text{loss}}_{\text{field},t}$</td>
<td>Average heat lost in solar collectors during time $(t - 1, t]$</td>
<td>$[0, \infty]$</td>
<td></td>
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<tr>
<td>$Q^{\text{loss}}_{\text{pipe},t}$</td>
<td>Average heat lost in piping during time $(t - 1, t]$</td>
<td>$[0, \infty]$</td>
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<tr>
<td>$Q_{r,t}$</td>
<td>Average heat removed from HTF during time $(t - 1, t]$</td>
<td>$[0.0, 10.0]$</td>
<td>[MW / MW]</td>
</tr>
<tr>
<td>$Q^{\dagger}_{r,t}$</td>
<td>Typical heat removed in Rankine cycle</td>
<td>$[0.0, 10 \Lambda]$</td>
<td>[MW]</td>
</tr>
<tr>
<td>$W_{t}$</td>
<td>Average useful work produced during time $(t - 1, t]$</td>
<td>$[0.0, \Lambda]$</td>
<td>MW</td>
</tr>
<tr>
<td>$W^{\dagger}_{t}$</td>
<td>Typical average useful work</td>
<td>$[0.0, \Lambda]$</td>
<td>MW</td>
</tr>
<tr>
<td>$w_{s,t}$</td>
<td>Average solar field pump duty during time $(t - 1, t]$</td>
<td>$[0, \infty]$</td>
<td>MW</td>
</tr>
<tr>
<td>$x_{t}$</td>
<td>Slack variable for steam cycle work correlation</td>
<td>varies</td>
<td>MW</td>
</tr>
</tbody>
</table>

We assume that, when a storage tank is full, it cools down one °C in 24 hours with an average tank temperature of 400 °C. This gives the global heat transfer coefficient:

$$UA = \frac{(M_{1,0} + M_{2,0})(1 \, ^{\circ}\text{C}) C_p}{\zeta_{\text{tonne}}(400 \, ^{\circ}\text{C} - T_{\text{amb}})(24 \, \text{hr}) c_s hr}.$$  \hspace{1cm} (A.6)

This coefficient depends on the total mass of heat transfer fluid in the storage system.

B Detailed Mathematical Models

This appendix presents the full resolution physical model for a parabolic trough CSP systems, organized into four subsystems: solar collectors, steam cycle, thermal energy storage, and operational restrictions. Then, for completeness, the linearized surrogate models used in (AR1) are presented. Finally, formulations (P2.1), (P2.2), and (P3) are restated with references to the full model equations.
Solar Collectors

Heat collected in the solar field is modeled by using an energy balance and correlations for the field (i.e., collector) heat losses \( Q_{\text{field,}t}^{\text{loss}} \) and pipe heat losses \( Q_{\text{pipe,}t}^{\text{loss}} \) developed in \(^{27}\) from data for the SEGS VI solar field. This gives the following set of nonlinear equations:

\[
C_p \dot{m}_{s,t} (T_s,t - T_2,t) \leq \eta_{\text{field}}^{\text{solar}} A_{\text{solar}} \Theta \left( \theta_t - \frac{Q_{\text{field,}t}^{\text{loss}}}{\omega_{\text{mirror}}} - Q_{\text{pipe,}t}^{\text{loss}} \right), \quad T_s,t \geq T_2,t + 1 ^\circ C \tag{B.7}
\]

\[
f_1(T_{\text{out}}, T_{\text{in}}) := \frac{\alpha_0 (T_{\text{out}} - T_{\text{in}}) + \frac{\alpha_1}{2} (T_{\text{out}}^2 - T_{\text{in}}^2) + \frac{\alpha_2}{3} (T_{\text{out}}^3 - T_{\text{in}}^3) + \frac{\alpha_3}{4} (T_{\text{out}}^4 - T_{\text{in}}^4)}{T_{\text{out}} - T_{\text{in}}}
+ \frac{T_{\text{out}} - T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}} \left[ \beta_0 (T_{\text{out}} - T_{\text{in}}) + \frac{\beta_1}{3} (T_{\text{out}}^3 - T_{\text{in}}^3) \right]
\]

\[
Q_{\text{field,}t}^{\text{loss}} = f_1(T_{s,t}, T_{2,t}) \tag{B.9}
\]

\[
f_2(\Delta T) := 0.01693 \Delta T - 0.0001683 \Delta T^2 + 6.78 \cdot 10^{-7} \Delta T^3 \tag{B.10}
\]

\[
f_3(T_{\text{out}}, T_{\text{in}}) := \frac{T_{\text{out}} + T_{\text{in}}}{2} - T_{\text{amb}} \tag{B.11}
\]

\[
Q_{\text{pipe,}t}^{\text{loss}} = f_2(f_3(T_{s,t}, T_{2,t})) \tag{B.12}
\]

Here, \( t \in \mathcal{T}_{\text{sun}} := \{ t \in \mathcal{T} : \theta_t \geq \theta_{\text{min}} \} \) is the set of times with acceptable solar irradiance, \( C_p \) is the HTF heat capacity (2.4 kJ/kg\(^\circ\)C), \( \eta_{\text{field}}^{\text{solar}} \) is the effective field efficiency (71.33\%), \( A_{\text{solar}} \) is the collector area (182,000 m\(^2\)), \( \Theta \) is the solar area multiplier, \( \theta_t \) is the horizontal solar irradiance at time \( t \), \( T_{\text{amb}} \) is the ambient temperature (25 \(^\circ\)C), \( \omega \) is the mirror width (5 m), \( \theta_{\text{min}} \) is a cutoff solar irradiance (50 W/m\(^2\)), and parameters \( \alpha \) and \( \beta \) are given in Table 6. We set \( Q_{\text{field,}t}^{\text{loss}} = Q_{\text{pipe,}t}^{\text{loss}} = \dot{m}_{s,t} = 0 \) for all \( t \in \mathcal{T}^* \setminus \mathcal{T}_{\text{sun}} \).

The pressure drop through the solar field and associated pumping work are calculated as:

\[
\Delta P_{s,t} = \kappa \bar{c}^{\text{Pa}} \left( \dot{m}_{s,t} \Theta^{-1} \right)^2 \tag{B.13}
\]

\[
w_{s,t} = \frac{\bar{c}^{\text{bar}}}{\eta_{\text{solar}}^{\text{pump}} \rho} \frac{\Delta P_{s,t} \dot{m}_{s,t}}{\Delta T_{s,t}} \tag{B.14}
\]
for all $t \in T_{\text{sun}}$. The field pressure drop coefficient ($\kappa$) is obtained from.\textsuperscript{27} We set $\Delta P_{t,s} = W_{s,t} = 0$ for all $t \in T^* \setminus T_{\text{sun}}$. To simplify the model, $\dot{m}_{s,t}$ is replaced with its upper bound in equation (B.13). This substitution fixes $\Delta P_{s,t}$ and makes (B.14) a linear constraint.

**Steam Cycle**

The power block is modeled using correlations for gross work ($W_t$) and for the HTF return temperature ($T_{r,t}$) obtained from.\textsuperscript{27} These correlations were constructed by regressing detailed simulations of the SEGS VI power plant and are given by:

\begin{equation}
 f_4(\dot{m}, T, P) := \gamma_0 + \gamma_1 \dot{m} + \gamma_2 \dot{m}^2 + \gamma_3 P + \gamma_4 T + \gamma_5 T^2 + \gamma_6 \dot{m} P + \gamma_7 \dot{m} T + \gamma_8 P T,
\end{equation}

\begin{equation}
 W_t = f_4(\dot{m}_{s,t}, T_{1,t}, P_{c,t}) + x_t
\end{equation}

\begin{equation}
 f_5(\dot{m}, T) := \tau_0 + \tau_1 \dot{m} + \tau_2 \dot{m}^2 + \tau_3 T + \tau_4 T^2 + \tau_5 \dot{m} T,
\end{equation}

\begin{equation}
 T_{r,t} = f_5(\dot{m}_{t}, T_{1,t}) + z_t
\end{equation}

for all $t \in T^*$. Here, $P_{c,t}$ is the operating pressure of the condenser and is a free variable bounded according to.\textsuperscript{27} Parameters $\gamma$ and $\tau$ are given in Table 7. The correlations in (B.15) and (B.17) only apply when the power plant is in ON mode. Consequently, (B.16) and (B.18) are relaxed with slack variables $x_t$ and $z_t$.

We use $Q_{r,t}$ to represent the heat removed from the HTF in the Rankine cycle during time $(t - 1, t]$. This variable is computed using the energy balance:

\begin{equation}
 Q_{r,t} = C_p \zeta_M^k \dot{W}_{r,t} (T_{\text{avg}}_{1,t} - T_{r,t})
\end{equation}

The power block efficiency (given by $W_t/Q_{r,t}$) is implicit in the model. The Rankine cycle and pumping duties are coupled to net energy ($E$) and onsite energy ($\hat{E}$) in the market participation model.
Thermal Energy Storage

The storage tanks are modeled using dynamic mass and energy balances in which $M_{i,t}$ and $T_{i,t}$ denote the HTF mass holdup and temperature in tank $i$ at time $t$. This gives the following set of equations:

\[ M_{1,t} = M_{1,t-1} + \Delta t_3 \zeta_{\text{tonne}}^{kg} (\dot{m}_{s,t} - \dot{m}_{r,t}) \]  
(B.21)

\[ M_{2,t} = M_{2,t-1} + \Delta t_3 \zeta_{\text{tonne}}^{kg} (\dot{m}_{r,t} - \dot{m}_{s,t}) \]  
(B.22)

\[ T_{i,t}^{\text{avg}} = 0.5T_{i,t} + 0.5T_{i,t-1}, \quad i \in \{1, 2\} \]  
(B.23)

\[ T_{1,t}M_{1,t} = T_{1,t-1}M_{1,t-1} + \Delta t_3 \zeta_{\text{tonne}}^{kg} \left( \dot{m}_{s,t}T_{s,t} - \dot{m}_{r,t}T_{1,t}^{\text{avg}} \right) - \Delta t_3 \zeta_{\text{MW}}^{kg} (C_p)^{-1} Q_{i,t}^{\text{loss}} \]  
(B.24)

\[ T_{2,t}M_{2,t} = T_{2,t-1}M_{2,t-1} + \Delta t_3 \zeta_{\text{tonne}}^{kg} \left( \dot{m}_{r,t}T_{r,t} - \dot{m}_{s,t}T_{2,t}^{\text{avg}} \right) - \Delta t_3 \zeta_{\text{MW}}^{kg} (C_p)^{-1} Q_{2,t}^{\text{loss}} \]  
(B.25)

\[ Q_{i,t}^{\text{loss}} = (U A)^{\text{MW}} (T_{i,t}^{\text{avg}} - T_{\text{amb}}), \quad i \in \{1, 2\} \]  
(B.26)

for all $t \in T^*$. Here, $\Delta t_3$ is the time step (5 minutes), which is the time elapsed between instants $t - 1$ and $t$. We use $T_{i,t}^{\text{avg}}$ to denote the average temperature in the storage tanks between time instants $t - 1$ and $t$. $Q_{i,t}^{\text{loss}}$ is the rate of heat loss from tank $i \in \{1, 2\}$ to the environment from $t - 1$ to $t$. This model can be derived by applying a midpoint formula (i.e., a modified Euler method) to the differential equations for the tank mass and energy balances and induces time coupling. This formula was chosen over others (e.g., forwards or backwards Euler method) to mitigate energy conservation errors.\(^{29}\) The model assumes the storage tanks are well-mixed. Bounds on $M_{i,t}$ and $T_{i,t}$ are given in Appendix A.

We approximate the total amount of energy in the storage system by using current operating conditions and an intermediate mass flow rate $\dot{\mu}^t$, typical values for useful work ($W_i^t$), return tem-
perature \((T^\dagger_{r,t})\), and heat removal \((Q^\dagger_{r,t})\). This gives:

\[
W^\dagger_t \leq f_4(\mu^\dagger_t, T_{avg}^1, P_{r,t}) \tag{B.27}
\]

\[
T^\dagger_{r,t} = f_5(\mu^\dagger_t, T_{avg}^1) \tag{B.28}
\]

\[
Q^\dagger_{r,t} = C_{p} \cdot kW \mu^\dagger_t (T_{avg}^1 - T^\dagger_{r,t}) \tag{B.29}
\]

We assume that the thermal efficiency stored above temperature \(T_{use}\) can be converted to electrical energy at the efficiency \(W^\dagger_t/Q^\dagger_{r,t}\). This is modeled with a smoothed max operator as follows:

\[
\max(x,0) \approx \tilde{\max}(x,0) := x + \sqrt{x^2 + \varepsilon^2}, \tag{B.30}
\]

\[
S_t = \frac{C_p \cdot kW \cdot \mu^\dagger_t}{\Lambda} \sum_{i \in \{1,2\}} \left(M_{i,t} \tilde{\max}(T_{i,t}^{avg} - T_{use}, 0)\right) \tag{B.31}
\]

Here, \(S_t\) (units of hours) approximates the energy stored in the system for electricity generation at full (nameplate) capacity. By using \(S_t\), we can ensure there is sufficient stored energy to satisfy simultaneous regulation up and spinning reserves dispatch for one hour. This is enforced by using the constraint:

\[
S_t \geq \sum_{\ell \in \mathcal{L}} A_{r,+ \ell,t} + A_{s,\ell,t}, \quad t \in T^* \tag{B.32}
\]

where \(A_{r,+ \ell,t}\) and \(A_{s,\ell,t}\) are allocated generation capacities for regulation up and spinning reserves, respectively, for market level \(\ell\). Finally, \(S_t\) is used in a terminal constraint to account for end of horizon time effects (e.g., \(S_0 = S_N\)). To facilitate implementation in a rolling horizon formulation, the following bounds are imposed at the end of each day \((t \in T_{eod})\):

\[
M_{1,t} \geq 0.05 (M_{1,0} + M_{2,0}), \quad M_{2,t} \geq 0.05 (M_{1,0} + M_{2,0}), \quad S_t \geq 0.5, \quad t \in T_{eod} \tag{B.33}
\]

Operational Limitations

The CSP plant can operate in three modes: \textbf{ON}, \textbf{SPIN} (spinning), and \textbf{OFF}. These are modeled with two binary state variables, \(y_{e,t}\) and \(y_{sp,t}\), which are specified over a 1-hour time discretization (defined by
the set $T_1$). We note that $y_{e,t} = 1$ implies ON mode, $y_{sp,t} = 1$ implies SPIN mode and $y_{e,t} = y_{sp,t} = 0$ implies OFF mode. Only one mode is permitted at a time, which is modeled using the constraint:

$$y_{e,t} + y_{sp,t} \leq 1, \quad t \in T_1$$  \hfill (B.34)

The binary variables are used to enforce (or relax) CSP physics and operational restrictions specific to individual modes (discussed below). When in ON mode, the electricity generated by the Rankine cycle must be above the minimum operating capacity $\lambda$:

$$W_t \geq \Lambda \lambda y_{e,t}$$  \hfill (B.35)

When in OFF or SPIN mode ($y_{e,t} = 0$), the Rankine cycle is not generating electricity and $W_t$ must be set to zero:

$$W_t \leq \Lambda y_{e,t}.$$  \hfill (B.36)

Moreover, under this mode the correlation (B.15) is only valid for $\dot{m}_{r,t} \in [\mu_{ON}, \bar{\mu}]$:

$$\dot{m}_{r,t} \geq \mu_{ON} y_{e,t}$$  \hfill (B.37)

$$\dot{m}_{r,t} \leq \bar{\mu} (y_{e,t} + y_{sp,t})$$  \hfill (B.38)

The market model (presented in detail in$^{13}$) can be configured to only allow for energy purchases when $y_{e,t} = 0$. This is important, as a CSP plant may need to run the solar field pumps with the power block in OFF mode. When the power block is ON, however, only electricity should be sold.

In SPIN mode ($y_{sp,t} = 1$), the steam turbines remain spinning and ready to begin electricity generation within 10 minutes of dispatch. This requires a small amount of heat from thermal storage, which modeled as $\Lambda \xi$. The minimum HTF mass flowrate, $\mu_{SPIN}$, is incorporated as:

$$\dot{m}_{r,t} \geq \mu_{SPIN} y_{sp,t}$$  \hfill (B.39)

Correlation (B.17) does not hold when in SPIN mode and thus the slack variable $z_t$ must be relaxed:

$$\mu_{z} y_{sp,t} \leq z_t \leq \bar{\mu}_{z} y_{sp,t}$$  \hfill (B.40)
where \( \mu_z \) and \( \bar{\mu}_z \) are “big-M” parameters (given in Appendix). Moreover, \( T_{r,t} \) cannot be directly computed for spinning mode. The HFT temperature difference across the Rankine cycle is bounded with \( \Delta T_{\text{SPIN}} = 50 \text{ }^\circ\text{C} \) and \( \Delta \bar{T}_{\text{SPIN}} = 200 \text{ }^\circ\text{C} \) as:

\[
\Delta T_{\text{SPIN}} y_{sp,t} \leq T_{1,t} - T_{r,t} \leq \Delta \bar{T}_r + (\Delta \bar{T}_r - \Delta \bar{T}_{\text{SPIN}}) y_{sp,t} \tag{B.41}
\]

where \( \Delta \bar{T}_r \) is the maximum temperature difference determined by the bounds for \( T_{1,t} \) and \( T_{r,t} \). Finally, \( y_{sp,t} = 1 \) implies \( Q_{r,t} = \xi \Lambda \) and thus:

\[
\Lambda \xi y_{sp,t} \leq Q_{r,t} \leq \bar{Q}_{r,t} - (\bar{Q}_{r,t} - \Lambda \xi) y_{sp,t}, \tag{B.42}
\]

where \( \bar{Q}_{r,t} \) is the upper bound for \( Q_{r,t} \).

We relax the power block correlations (B.15) by using slack variable \( x_t \) in (B.16). This is done by using the following logic:

**ON Mode:** \( y_{e,t} = 1 \Rightarrow y_{sp,t} = 0 \Rightarrow x_t = 0 \)

**SPIN Mode:** \( y_{sp,t} = 1 \Rightarrow y_{e,t} = 0 \Rightarrow \chi_{\text{SPIN}} \leq x_t \leq \bar{\chi}_{\text{SPIN}} \) \tag{B.43}

**OFF Mode:** \( y_{e,t} = 0 \) & \( y_{sp,t} = 0 \Rightarrow \chi_{\text{OFF}} \leq x_t \leq \bar{\chi}_{\text{OFF}} \)

where \( \chi_{\text{SPIN}} \), \( \bar{\chi}_{\text{SPIN}} \), \( \chi_{\text{OFF}} \), \( \bar{\chi}_{\text{OFF}} \) are determined by minimizing/maximizing (B.16) subject to variable bounds for SPIN and OFF modes. Equations (B.43) are then reformulated as:

\[
\chi_{\text{SPIN}} y_{sp,t} + \chi_{\text{OFF}} (1 - y_{e,t} - y_{sp,t}) \leq x_t \leq \bar{\chi}_{\text{SPIN}} y_{sp,t} + \bar{\chi}_{\text{OFF}} (1 - y_{e,t} - y_{sp,t}), \quad t \in \mathcal{T}. \tag{B.44}
\]

We also consider startup and shutdown schedule restrictions from\(^{11}\) To do so, we define \( \tau_{\text{ON}} \) as the minimum number of hours that the CSP must remain in ON mode after startup (i.e., when \( y_e \) is switched from 0 to 1). The operating schedule is constrained as follows:

\[
\sum_{j=t}^{t+\tau_{\text{ON}}-1} y_{e,j} \geq \tau_{\text{ON}} (y_{e,t} - y_{e,t-1}), \quad t \in \{I_{\text{ON}} + 1, ..., |\mathcal{T}_1^*| - \tau_{\text{ON}} + 1\} \tag{B.45}
\]
where \( I_{ON} \) is the number of hours the CSP must remain on due to the initial status and \( |T_1^*| \) is the length of the planning horizon. Shutdown (i.e., when \( y_e \) switched from 1 to 0) is constrained as:

\[
\sum_{j=t}^{t+\tau_{OFF}-1} (1 - y_{e,j}) \geq \tau_{OFF} \left( y_{e,t-1} - y_{e,t} \right), \quad t \in \{I_{OFF} + 1, \ldots, |T_1^*| - \tau_{OFF} + 1\}
\]

where \( \tau_{OFF} \) is the minimum number of hours a CSP plant must remain in OFF mode after shutdown and \( N_{OFF} \) is the number of hours the plant must initially remain in OFF mode due to the initial status. These constraints reduce wear-and-tear stress effects by preventing aggressive switching between modes. Equations (B.45) and (B.46) can be extended to explicitly restrict transition to and from SPIN mode. Additional constraints obtained from\(^{11}\) for beginning and end-of-horizon effects were considered but are omitted here for brevity. Alternative models, such as those proposed in,\(^{19}\) are also compatible with this framework.

We also require that \( \dot{m}_{r,t} \leq \mu_{start} \) at the (5-minute) timestep immediately after start-up and \( \dot{m}_{r,t} \leq \mu_{stop} \) during the (5-minute) timestep immediately before shutdown:

\[
\dot{m}_{r,t} \leq \mu_{start} + (\bar{\mu}_{ON} - \mu_{start}) (1 - y_{e,t} + y_{e,t-1}), \quad t \in T^*, \quad (B.47)
\]

\[
\dot{m}_{r,t} \leq \mu_{stop} + (\bar{\mu}_{ON} - \mu_{stop}) (1 + y_{e,t} - y_{e,t-1}), \quad t \in T^*. \quad (B.48)
\]

These constraints capture the ramping limits for the Rankine cycle.

**Dynamic Mixed Integer Optimization Formulation**

The above equations are used in the full resolution mixed integer nonlinear program (MINLP), which is referred to as (P1) in the text:
maximize \quad \text{Market Revenue}
\begin{align*}
s.t. \quad & \text{Market Model} \quad \text{See}^{13} \\
& \text{Solar Collector Model} \quad (B.7) - (B.14) \\
& \text{Steam Cycle Model} \quad (B.15) - (B.20) \\
& \text{Thermal Storage Model} \quad (B.21) - (B.33) \\
& \text{Operational Restrictions} \quad (B.34) - (B.48).
\end{align*}

Decision variables include the market participation schedule \((E_{\ell,t}, E_{\ell,t}^a, A_{a,\ell,t})\), the operating mode schedule \((y_{e,t}, y_{sp,t}, y_{n,t})\), and mass and energy flows in the CSP system \((E_{i,t}, M_{i,t}, Q_{s,t}, W_t)\) and so on).

**Linear Surrogate Model**

We now present the complete surrogate CSP physical model, starting with the thermal energy storage subsystem. We use \(S_t\) to represent the total energy in all of the storage tanks and consider an overall energy balance:

\[
S_t = S_{t-1} + \Delta t_3 (Q_{s,t} - Q_{r,t}), \quad t \in T
\]  

(B.49)

where \(S_0\) is computed from the initial tank mass holdups and temperatures using (B.31). The previously introduced mass balance equations for each storage tank, (B.24) and (B.25), are also used in the simplified CSP physics model. Variable \(E_{i,t}\) is introduced to model the energy holdups in each storage tank directly and new energy balances are derived:

\[
E_{1,t} = E_{1,t-1} + \Delta t_3 \left( C_p \frac{kW}{\text{MW}} \left[ \dot{m}_{s,t} \bar{T}_{2,t}^\text{avg} - \dot{m}_{r,t} \bar{T}_{1,t}^\text{avg} \right] + \Lambda Q_{s,t} - \bar{Q}_{1,t}^\text{loss} \right), \quad (B.50)
\]

\[
E_{2,t} = E_{2,t-1} + \Delta t_3 \left( C_p \frac{kW}{\text{MW}} \left[ \dot{m}_{r,t} \bar{T}_{1,t}^\text{avg} - \dot{m}_{s,t} \bar{T}_{2,t}^\text{avg} \right] - \Lambda Q_{r,t} - \bar{Q}_{2,t}^\text{loss} \right), \quad (B.51)
\]

with the initial energy state values defined as follows:

\[
E_{i,0} := T_{i,0} M_{i,0} \frac{C_p}{\text{MW}h} \frac{\zeta_{tonne}}{\zeta_{kg}}, \quad i \in \{1, 2\}.
\]  

(B.52)
The useful energy (above $T_{use}$) is calculated using $\tilde{T}_{i,t}^{avg}$ as follows:

$$S_t = \Lambda^{-1} \frac{\tilde{W}_t^\dagger}{Q_{r,t}} \sum_{i \in \{1, 2\}} \left( E_{i,t} \frac{\max(\tilde{T}_{i,t}^{avg} - T_{use}, 0)}{\tilde{T}_{i,t}^{avg}} \right). \quad (B.53)$$

A key issue is defining the simplified CSP physical model for the first iteration, when detailed enthalpy profiles are not available. In this case, (B.50), (B.51), (B.5), and (B.53) are not applicable. We thus derive an alternative surrogate model. We use $h_{1,t} \geq 0$ to represent the enthalpy that leaves tank 1 and enters tank 2 and note that this does not include the enthalpy sent from tank 1 to the Rankine cycle. Similarly, we let $h_{2,t} \geq 0$ represent the enthalpy that leaves tank 2 and enters tank 1 and note that this does not include the enthalpy gain from the solar field. Using these definitions, we can write $h_{1,t}$ and $h_{2,t}$ in terms of tank and flow rate variables:

$$h_{1,t} := C_p \zeta_{MW} \Lambda^{-1} \dot{m}_{r,t} T_{r,t} = C_p \zeta_{MW} \Lambda^{-1} \dot{m}_{r,t} T_{1,t}^{avg} - Q_{r,t} \quad (B.54)$$

$$h_{2,t} := C_p \zeta_{MW} \Lambda^{-1} \dot{m}_{s,t} T_{2,t}^{avg} = C_p \zeta_{MW} \Lambda^{-1} \dot{m}_{s,t} T_{s,t} - Q_{s,t} \quad (B.55)$$

We consider energy balances around the tanks:

$$E_{1,t} = E_{1,t-1} + \Delta t_3 C_p \zeta_{MW} (\dot{m}_{s,t} T_{s,t} - \dot{m}_{r,t} T_{1,t}^{avg}) \quad (B.56)$$

$$E_{2,t} = E_{2,t-1} + \Delta t_3 C_p \zeta_{MW} (\dot{m}_{r,t} T_{r,t} - \dot{m}_{s,t} T_{2,t}^{avg}) \quad (B.57)$$

We substitute the definitions (B.54) and (B.55) into the energy balances (B.56) and (B.57). Rearrangement yields the following energy balances:

$$E_{1,t} = E_{1,t-1} + \Delta t_3 \Lambda ((h_{2,t} + Q_{s,t}) - (h_{1,t} + Q_{r,t})) \quad (B.58)$$

$$E_{2,t} = E_{2,t-1} + \Delta t_3 \Lambda (h_{1,t} - h_{2,t}) \quad (B.59)$$

We propagate temperature bounds through definitions (B.54) and (B.55) to develop the following inequalities:

$$\dot{m}_{s,t} T_2 C_p \zeta_{MW} \leq \Lambda h_{2,t} \quad (B.60)$$
\[ \dot{m}_{s,t} \bar{T}_1 \, C_p \, \zeta_{MW} \geq \Lambda(h_{2,t} + Q_{s,t}) \]  
(B.61)

\[ \dot{m}_{s,t} \bar{T}_2 \, C_p \, \zeta_{MW} \geq \Lambda h_{2,t} \]  
(B.62)

\[ \dot{m}_{r,t} \bar{T}_1 \, C_p \, \zeta_{MW} \geq \Lambda(h_{1,t} + Q_{r,t}) \]  
(B.63)

\[ \dot{m}_{r,t} \bar{T}_1 \, C_p \, \zeta_{MW} \leq \Lambda(h_{1,t} + Q_{r,t}) \]  
(B.64)

where \( \bar{T}_1, \bar{T}_1, \bar{T}_2, \bar{T}_3 \) represent the upper and lower temperature bounds for storage tanks 1 and 2, respectively. As in the case of (0.5), we link mass and energy flows using temperature bounds of the form:

\[ M_{i,t} \bar{T}_i \leq E_{i,t} \, C^{-1}_p \, \zeta_{MW} \, \zeta_{tonne} \leq M_{i,t} \bar{T}_i, \quad i \in \{1, 2\}, \quad t \in T^* \]  
(B.65)

Using a similar approach, we now derive the simplified model for the solar collector field. Bounds for heat collected is solar collector, \( Q_{s,t} \), are calculated from solar irradiance, \( \theta_t \), and (fixed) heat loss rates \( \bar{Q}_{loss field} \) and \( \bar{Q}_{loss pipe} \) obtained at the solution of (P3) at the previous iteration \( k - 1 \):

\[ Q_{s,t} \leq \left( \theta_t - \frac{\bar{Q}_{loss field}}{\omega} - \frac{\bar{Q}_{loss pipe}}{\omega} \right) \zeta_{kW} \, \zeta_{field} \, \zeta_{solar} \, \zeta_{field} \, \zeta_{solar} \, A_{solar} \, \zeta_{field} \]  
(B.66)

This heat rate is calculated using a mass balance and fixed solar collector inlet and outlet temperatures from (P3):

\[ \Lambda Q_{s,t} = C_p \zeta_{MW} \left( \bar{T}_{s,t} - \bar{T}_{1,t} \right) \dot{m}_{s,t} \]  
(B.67)

The mass flowrate through the solar collector, \( \dot{m}_{s,t} \), is bounded using results from (P3):

\[ (1 - \delta_m) \bar{m}_{s,t} \leq \dot{m}_{s,t} \leq (1 + \delta_m) \bar{m}_{s,t} \]  
(B.68)

where \( 0 \leq \delta_m \) is a tunable trust-region parameter.

Pressure drops through the solar collector are fixed as follows:

\[ \Delta \bar{P}_{s,t} = \kappa \zeta_{bar} \left( \bar{m}_{s,t} \zeta^{-1} \right)^2 \]  
(B.69)
which yields a linear constraint for solar field pump work of the form:

\[
    w_{s,t} = \frac{\Delta \bar{P}_{s,t} \bar{\zeta}_{bar} \eta_{solarpump} \tilde{m}_{s,t}}{\rho} \quad \text{(B.70)}
\]

Similar to the case of the storage tank model, special care is required for the first iteration. Here, heat loss rates are approximated as follows:

\[
    \bar{Q}_{\text{field},t}^{\text{loss}} = f(250 \, ^\circ \text{C}, 400 \, ^\circ \text{C}, \theta_t), \quad \bar{Q}_{\text{pipe},t}^{\text{loss}} = f(300 \, ^\circ \text{C}) \quad \text{(B.71)}
\]

Due to mismatch between the full-resolution CSP model, given by (B.7) - (B.12), the field inlet (\(\bar{T}_{1,t}\)) and outlet (\(\bar{T}_{s,t}\)) temperatures cannot be directly used in (B.67). Instead, the energy balance is approximated using lower (25 \(^\circ\)C) and upper (100 \(^\circ\)C) bounds for the temperature difference:

\[
    C_p \zeta_{MW}^{kW} (25 \, ^\circ \text{C}) \leq \Lambda \tilde{m}_{s,t} \leq C_p \zeta_{MW}^{kW} (100 \, ^\circ \text{C}) \quad \text{(B.72)}
\]

Finally, we derive a simplified Rankine cycle model. We linearize (B.15) around \(\dot{m}_{r,t}\) as:

\[
    f_{6,t}(\dot{m}_{r,t}) := \gamma_0 + \gamma_1 \dot{m}_{r,t} + \gamma_2 \dot{m}_{r,t} \bar{m}_{r,t} + \gamma_3 \bar{P}_{r,t} + \gamma_4 \bar{T}_{r,t} + \gamma_5 \bar{T}_{r,t}^2 + \gamma_6 \dot{m}_{r,t} \bar{P}_{r,t} + \gamma_7 \bar{m}_{r,t} \bar{T}_{r,t} + \gamma_8 \bar{P}_{r,t} \bar{T}_{r,t}, \quad \text{(B.73)}
\]

where \(\bar{P}_{r,t}, \bar{T}_{r,t}, \text{and} \bar{m}_{r,t}\) are obtained from (P3). The function \(f_{6}(\cdot)\) is used to calculate \(W_t\) from:

\[
    W_t + x_t = f_{6,t}(\bar{m}_{r,t}) \quad \text{(B.74)}
\]

where \(x_t\) is a slack variable that relaxes according to the following logic:

\[
    \chi_{0,t} := f_{6,t}(0.0), \quad \chi_t := \min (\chi_{0,t}, f_{6,t}(\bar{\mu}_{0t})), \quad \bar{\chi}_t := \max (\chi_{0,t}, f_{6,t}(\bar{\mu}_{0t})), \quad \text{(B.75)}
\]

\[
    y_{e,t} = 1 \implies y_{sp,t} = 0 \implies x_t = 0
\]

\[
    y_{sp,t} = 1 \implies y_{e,t} = 0 \implies \chi \leq x_t \leq \bar{\chi} \quad \text{(B.76)}
\]

\[
    y_{e,t} = 0, \ y_{sp,t} = 0 \implies x_t = \chi_{0,t}
\]

which is translated into the following constraint:

\[
    (\chi_t - \chi_{0,t}) y_{sp,t} + \chi_{0,t} (1 - y_{e,t}) \leq x_t \leq (\bar{\chi}_t - \chi_{0,t}) y_{sp,t} + \chi_{0,t} (1 - y_{e,t}), \quad t \in \mathcal{T}^* \quad \text{(B.77)}
\]
If \texttt{SPIN} mode is not permitted, the model can be simplified as:

\[ \chi_t = \bar{\chi}_t = \chi_{0,t}, \quad (B.78) \]

and \((B.77)\) is replaced with an equality constraint. The heat transferred from the HTF loop into the Rankine cycle is calculated using temperatures from (P3):

\[
\Lambda Q_{r,t} = C_p \zeta_{MW}^{kW} (\bar{T}_{1,t} - \bar{T}_{r,t}) (\bar{m}_{r,t} + g_t), \quad t \in T^* \quad (B.79)
\]

where \(g_t\) is a slack variable used to relax the equation when \texttt{SPIN} mode is active. We recall that, during \texttt{SPIN} mode, the temperature difference between HTF entering and leaving the Rankine cycle is bounded between \(\Delta T_{\text{SPIN}} = 50 \, ^\circ\text{C}\) and \(\Delta \bar{T}_{\text{SPIN}} = 200 \, ^\circ\text{C}\). This fact is used to derive upper and lower bounds for \(g_t\). We substitute \(Q_{r,t} = \xi\) into \((B.79)\) and use \(\sigma\) to represent the mass flowrate through the Rankine cycle:

\[
\sigma_t = \frac{\xi \Lambda}{(\bar{T}_{1,t} - \bar{T}_{r,t}) C_p \zeta_{MW}^{kW}} \quad (B.80)
\]

We relax the temperature difference to be between \(\Delta \bar{T}_{\text{SPIN}}\) and \(\Delta T_{\text{SPIN}}\). We use \(\bar{\sigma}_t\) and \(\sigma_t\) to represent lower and upper bounds for the \(\bar{m}_{r,t}\) during \texttt{SPIN} mode:

\[
\bar{\sigma}_t := \frac{\xi \Lambda}{\Delta \bar{T}_{\text{SPIN}} C_p \zeta_{MW}^{kW}}, \quad \sigma_t := \frac{\xi \Lambda}{\Delta T_{\text{SPIN}} C_p \zeta_{MW}^{kW}} \quad (B.81)
\]

We use \(\sigma_t\), \(\bar{\sigma}_t\), and \(\sigma_t\) to relax \(g_t\) during \texttt{SPIN} mode as:

\[
(\sigma_t - \bar{\sigma}_t) y_{sp,t} \leq g_t \leq (\sigma_t - \sigma_t) y_{sp,t}, \quad t \in T \quad (B.82)
\]

An alternative model for the first iteration is required here as well. For this case, a constant heat-to-electricity efficiency, \(\eta_r\), is assumed. This gives,

\[
\Lambda \eta_r Q_{r,t} \geq W_t. \quad (B.83)
\]

If spinning reserves are not considered, then \((B.83)\) is posed as an equality constraint.
Mass and energy flows are linked by bounding the HTF temperature difference across the Rankine cycle between 35 °C and 110 °C.

\[ C_p \eta_r \zeta_{MW}^kW (35 \degree C) \dot{m}_{s,t} \leq W_t + \xi \Lambda y_{sp,t} \]  
\[ W_t \leq C_p \eta_r \zeta_{MW}^kW (110 \degree C) \dot{m}_{s,t} \]  

(B.84)  

(B.85)

**Scheduling Subproblem Formulations**

In summary, the above equations are used to form two versions of (P2). For the first iteration of (AG1) we consider the following scheduling problem (P2.1):

\[
\begin{align*}
\text{maximize} & \quad \text{Market Revenue} \\
\text{s.t.} & \quad \text{Market Model} \quad \text{See}^{13} \\
& \quad \text{Thermal Storage Model} \quad (B.24), (B.25), (B.49), (B.52), (B.58) - (B.65) \\
& \quad \text{Solar Collector Model} \quad (B.68) - (B.72) \\
& \quad \text{Rankine Cycle Model} \quad (B.35), (B.36) - (B.39), (B.83) - (B.85) \\
& \quad \text{Operational Restrictions} \quad (B.34) - (B.39), (B.42) - (B.48). 
\end{align*}
\]

For subsequent iterations we consider the scheduling problem (P2.2):

\[
\begin{align*}
\text{maximize} & \quad \text{Market Revenue} \\
\text{s.t.} & \quad \text{Market Model} \quad \text{See}^{13} \\
& \quad \text{Thermal Storage Model} \quad (B.24), (B.25), (B.49) - (B.53) \\
& \quad \text{Solar Collector Model} \quad (B.66) - (B.70) \\
& \quad \text{Rankine Cycle Model} \quad (B.35), (B.36) - (B.39), (B.73) - (B.82) \\
& \quad \text{Operational Restrictions} \quad (B.34) - (B.39), (B.42) - (B.48). 
\end{align*}
\]

**Dynamic Optimization Subproblem Formulation**

Using the equations described in this appendix, problem (P3) is given by:
maximize Market Revenue

\[ \text{s.t.} \]

Market Model \hspace{1cm} \text{See}\^{13}

Solar Collector Model \hspace{1cm} (B.7) - (B.14)

Steam Cycle Model \hspace{1cm} (B.15) - (B.20)

Thermal Storage Model \hspace{1cm} (B.21) - (B.33)

with \( y_{e,t} \) and \( y_{sp,t} \) fixed.

References

[1]


[20] HSL. A collection of fortran codes for large scale scientific computation.


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Figure 3: Revenues for (P2) and (P3) over 25 iterations.

(a) $\delta_T = 10 \, ^\circ C$

(b) $\delta_T = 30 \, ^\circ C$
Figure 4: Computational times for (P2) and (P3) over 25 iterations.

(a) $\delta_T = 10^\circ C$

(b) $\delta_T = 30^\circ C$
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<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAM</td>
<td>31.2</td>
<td>9.2</td>
</tr>
<tr>
<td>FFM</td>
<td>29.7</td>
<td>33.5</td>
</tr>
<tr>
<td>RTD</td>
<td>29.9</td>
<td>64.0</td>
</tr>
<tr>
<td>DAM minus FFM</td>
<td>1.5</td>
<td>25.0</td>
</tr>
<tr>
<td>DAM minus RTD</td>
<td>1.4</td>
<td>41.9</td>
</tr>
</tbody>
</table>
Table 4: Revenues and computational times for (P2) and (P3) over 25 iterations.

<table>
<thead>
<tr>
<th>$\delta_T$</th>
<th>Revenue</th>
<th>Total Time for (P2)</th>
<th>Total Time for (P3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 °C</td>
<td>3757 $/week</td>
<td>0.6 CPU-hours</td>
<td>0.3 CPU-hours</td>
</tr>
<tr>
<td>10 °C</td>
<td>4840 $/week</td>
<td>0.7 CPU-hours</td>
<td>1.6 CPU-hours</td>
</tr>
<tr>
<td>15 °C</td>
<td>4851 $/week</td>
<td>0.3 CPU-hours</td>
<td>6.1 CPU-hours</td>
</tr>
<tr>
<td>20 °C</td>
<td>4882 $/week</td>
<td>0.1 CPU-hours</td>
<td>2.7 CPU-hours</td>
</tr>
<tr>
<td>25 °C</td>
<td>4876 $/week</td>
<td>0.2 CPU-hours</td>
<td>1.2 CPU-hours</td>
</tr>
<tr>
<td>30 °C</td>
<td>4901 $/week</td>
<td>0.1 CPU-hours</td>
<td>0.6 CPU-hours</td>
</tr>
<tr>
<td>100 °C</td>
<td>4885 $/week</td>
<td>0.1 CPU-hours</td>
<td>1.7 CPU-hours</td>
</tr>
</tbody>
</table>
Table 5: Comparison of (AR1) with settings $\delta_T = 30 \, ^\circ C$ and 10 iterations and four algorithms in Bonmin: B-BB is branch-and-bound, B-QG is branch-and-cut, B-OA is outer approximation, and B-Hyb is a hybrid strategy. All five algorithms were tried for each instance. Omitted rows indicate failure to find a feasible solution within 8 hours.

<table>
<thead>
<tr>
<th>Market Products</th>
<th>Horizon</th>
<th>Algorithm</th>
<th>Objective</th>
<th>CPU-Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Only</td>
<td>1 day</td>
<td>(AR1)</td>
<td>$621.4$</td>
<td>44.3 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-BB</td>
<td>$621.4$</td>
<td>271.1 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-QG</td>
<td>$621.4$</td>
<td>111.7 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-Hyb</td>
<td>$621.4$</td>
<td>71.6 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-OA</td>
<td>$589.8$</td>
<td>15,976 s</td>
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<tr>
<td>Energy Only</td>
<td>2 days</td>
<td>(AR1)</td>
<td>$1231.5$</td>
<td>131.5 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-BB</td>
<td>$1256.5$</td>
<td>21,656.9 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-QG</td>
<td>$1255.4$</td>
<td>3611.7 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-Hyb</td>
<td>$1191.0$</td>
<td>3229.3 s</td>
</tr>
<tr>
<td>Energy Only</td>
<td>3 day</td>
<td>(AR1)</td>
<td>$1869.1$</td>
<td>333.3 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-BB</td>
<td>$1783.2$</td>
<td>21,760.6 s</td>
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<tr>
<td></td>
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<td>B-QG</td>
<td>$1886.3$</td>
<td>22,036.7 s</td>
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<td></td>
<td></td>
<td>B-Hyb</td>
<td>$1896.4$</td>
<td>7159.1 s</td>
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<tr>
<td>All Products</td>
<td>1 day</td>
<td>(AR1)</td>
<td>$670.9$</td>
<td>31.7 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-BB</td>
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<td>989.4 s</td>
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<td></td>
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<td>B-QG</td>
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<td>142.4 s</td>
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<td></td>
<td>B-Hyb</td>
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<td>244.9 s</td>
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<tr>
<td>All Products</td>
<td>2 days</td>
<td>(AR1)</td>
<td>$1369.8$</td>
<td>129.5 s</td>
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<td></td>
<td></td>
<td>B-BB</td>
<td>$1366.6$</td>
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<tr>
<td></td>
<td></td>
<td>B-QG</td>
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<td>21,780.5 s</td>
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<tr>
<td></td>
<td></td>
<td>B-Hyb</td>
<td>$1369.2$</td>
<td>4,543.8 s</td>
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Table 6: Coefficient values for (B.8), from\textsuperscript{27}

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Value</th>
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<tbody>
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Table 7: Coefficient values for (B.15) and (B.17), from\textsuperscript{27}

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<tr>
<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
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<tbody>
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