Space-Time Dynamics of Electricity Markets Incentivize Technology Decentralization

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Abstract

We study economic incentives provided by space-time dynamics of day-ahead and real-time electricity markets. Specifically, we seek to analyze to what extent such dynamics promote decentralization of technologies for generation, consumption, and storage (which is essential to obtain a more flexible power grid). Incentives for decentralization are also of relevance given recent interest in the deployment of small-scale modular technologies (e.g., modular ammonia and biogas production systems). Our analysis is based on an asset placement problem that seeks to find optimal locations for generators and loads in the network that minimize profit risk. We show that an unconstrained version of this problem can be cast as an eigenvalue problem. Under this representation, optimal network allocations are eigenvectors of the space-time price covariance matrix while the eigenvalues are the associated profit variances. We also construct a placement formulation that captures different risk metrics and constraints on types of technologies to systematically analyze trade-offs in expected profit and risk. Our analysis reveals that space-time market dynamics provide significant incentives for decentralization and strategic asset placement but that full mitigation of risk is only possible through simultaneous investment in generation and loads (which can be achieved using batteries or microgrids).

Keywords: space-time; dynamics; markets; decentralization

1 Introduction

Decentralization of technologies for power generation (e.g., power plants), consumption (e.g., manufacturing facilities and data centers), and storage (e.g., batteries) is an on going industrial trend [13, 14]. From the perspective of an independent system operator (ISO), decentralization is desirable as it can provide spatial flexibility to control network flows and to overcome limited transmission infrastructure [2, 9]. In addition, large centralized power generation and consumption facilities can become liabilities during extreme weather or cyber attacks [11]. To give an idea of the risk that large centralized facilities pose to the power grid, consider the fact that the load of a conventional ammonia manufacturing plant is around 64 MW [5] and that the load of a large data center reaches 50 MW [1]
(equivalent to the load of tens of thousands or homes). Similarly, the power supply of a large central-
ized power plant such as the Hammond plant in Georgia is 800 MW [6]. The growing demand from
large data centers is of particular concern as it is projected that, within the next decade, the loads
from such facilities will represent over 20% of the total grid load [9]. Another issue associated with
centralized facilities is that they provide limited investment flexibility to mitigate long-term risks in
electricity prices and policy. The need to mitigate investment risks is promoting the development and
deployment of smaller-scale (modular) technologies [7,10,12,16]. On the other hand, it is well-known
that large centralized systems benefit from economies of scale and thus a strong trade-off between
expected profit and risk exist.

![Figure 1: Electricity price fluctuation in RTM on February 5, 2015 in CAISO.](image)

Because electricity prices are a key driving factor in the revenue/cost of facilities, space-time price
fluctuations must be considered in investment and operating decisions. For instance, power gen-
eration and consumption facilities often sell/purchase electricity on the Day-Ahead Energy Market
(DAM) as opposed to the Real-Time Energy Market (RTM) to minimize risk, as the former is far less
volatile [3,4,8]. The temporal and spatial volatility of electricity prices in RTM is illustrated in Figure
1. Here, we show the nodal price change over 20 minutes for a specific day in California. We see
that, under a 20-minute period, the average electricity price jumps from 48.42 USD/MWh to 592.33
USD/MWh and then falls down again to 35.15 USD/MWh. Here, we also see that such fluctuations
are less abrupt at some network locations. Price volatility is less severe in day-ahead markets; in
fact, day-ahead markets are precisely designed to pre-allocate generation and loads in the network
in order to mitigate risk [17]. On the other hand, the average RTM price is typically lower than
the average DAM price. Consequently, there exists a premium to participate in the DAM (in order
to avoid RTM volatility and associated risk). This suggests that there exists an economic incentive
to decentralize (diversify) generation and load assets over multiple network locations in order to ex-
plot spatial differences in DAM and RTM prices (and with this avoid large premia). Similarly, spatial
variations in DAM and RTM prices can be exploited by decentralized facilities to maximize profit.
For instance, large cloud computing providers are currently placing data centers strategically in the network in order to avoid large electricity costs [9]. One could also envision that small modular manufacturing facilities can be relocated to exploit more favorable prices. A challenge that arises in this context is that the DAM and RTM prices exhibit complex spatio-temporal dynamics and correlation patterns [15]. As a result, it is non-trivial to identify suitable degrees of asset decentralization and optimal locations for such assets.

In this work, we propose a computational framework for analyzing economic incentives created by space-time dynamics of electricity markets. Our framework is based on an asset placement formulation that seeks to find optimal locations for generation and load (consumption) assets in the network that minimize profit risk. We show that an unconstrained version of this problem can be cast as an eigenvalue problem. Under this representation, optimal network allocations are eigenvectors of the space-time price covariance matrix, while the eigenvalues are the profit variances that result from such allocations. Consequently, risk analysis can be performed in a systematic and computationally efficient manner by using principal component analysis (PCA). Analysis using CAISO market data for 2015 reveals that there exists a large number of asset placement strategies that completely eliminate risk. We construct a constrained placement problem that captures constraints on the types of assets and that trade-offs risk and expected profit. Unfortunately, for the ISO-scale data sets of interest, this problem is a large-scale mixed-integer quadratic programming (MIQP) problem that cannot be solved with current solvers. We use the mean absolute deviation as an alternative risk measure to obtain a more scalable (but still challenging) mixed-integer linear program (MILP). Our analysis reveals that space-time market dynamics provide significant incentives for strategic diversification and asset placement but that complete mitigation of revenue risk is only possible by simultaneous investment in decentralized generation and load assets (which can also be achieved by using batteries or hybrid systems such as microgrids). These results are of relevance given the recent interest in the deployment of small-scale modular technologies.

The paper is structured as follows. In the following section, we motivate our discussion by conducting a basic space-time analysis of electricity markets in California. In Section 2 we formulate the technology placement problem, interpret it as an eigenvalue problem, and provide scalable constrained variants. A detailed analysis of the California ISO data set using the placement problem formulations is provided in Section 3.

2 Optimal Placement Problem

We capture the space-time price data set in a matrix \( \Pi \in \mathbb{R}^{m \times n} \). Here, the number of columns \( n \) is the number of spatial network locations (nodes) and the number of rows \( m \) is the number of time points. The price at the spatial location (network node) \( j \) is modeled as a random variable (denoted as \( P_j \)) and we use \( P = \{ P_1, ..., P_m \} \) to denote a random vector containing all node prices. Consequently, the matrix entry \( \Pi_{i,j} \) is interpreted as the \( i \)-th time realization of the price \( P_j \) and we assume that the probability of the realization is \( 1/m \) (i.e., we ignore temporal price correlations). We use \( p_i := \Pi_{i,:} \in \mathbb{R}^n, i = 1, ..., m \) to denote the realizations of the spatial price vector \( P \). We denote the set of spatial
locations as $\mathcal{N} := \{1, ..., n\}$ and the set of all time realizations as $\mathcal{M} := \{1, ..., m\}$. Moreover, prices have units of USD/MWh.

We define the sample average price vector $\mu \in \mathbb{R}^n$, which is given by:
\[
\mu := \frac{1}{m} \sum_{i \in \mathcal{M}} \Pi_i. \approx \mathbb{E}[P]. \tag{2.1}
\]

We use $\Sigma \in \mathbb{R}^{n \times n}$ to denote the sample price covariance matrix, which is given by:
\[
\Sigma := \frac{1}{m-1} \sum_{i \in \mathcal{M}} (p_i - \mu)(p_i - \mu)^T. \tag{2.2}
\]

This matrix is an approximation of the price covariance matrix $\mathbb{E}[(P - \mathbb{E}[P])(P - \mathbb{E}[P])^T]$. The entries of the sample covariance matrix are $\Sigma_{jk} = \frac{1}{m-1} \sum_{i \in \mathcal{M}} (\Pi_{i,j} - \mu_j)(\Pi_{i,k} - \mu_k)^T$ which are approximations of the covariance $\text{Cov}(P_j, P_k) = \mathbb{E}[(P_j - \mathbb{E}[P_j])(P_k - \mathbb{E}[P_k])]$ for $j, k \in \mathcal{N}$. The covariance matrices are symmetric and positive semi-definite.

### 2.1 Unconstrained Formulation and Eigenvalue Interpretation

Given the price data, we seek to identify optimal locations for loads and generators in the network that minimize profit variance (a standard risk measure). We define an allocation vector $w \in \mathbb{R}^n$ and the profit function $\phi(w, P) := w^T P = \sum_{j \in \mathcal{N}} w_j P_j$ and note that this is a random variable. We interpret a positive node allocation $w_j > 0$ as an injection of power (a generation asset incurring revenue for a positive price) and a negative node allocation $w_j < 0$ as a withdrawal of power (a load asset incurring cost for a positive price). The node allocations $w_j$ have units of MWh. If the prices are negative, a positive allocation incurs cost and a negative allocation incurs a revenue.

The sample average of the profit is given by:
\[
\mu_{\phi}(w) = \frac{1}{n} \sum_{i \in \mathcal{M}} \phi(w, p_i) \approx \mathbb{E}[\phi(w, P)] \tag{2.3}
\]
and its sample variance is
\[
\Sigma_{\phi}(w) = \frac{1}{m-1} \sum_{i \in \mathcal{M}} (\phi(w, p_i) - \mu_{\phi}(w))^2 \approx \mathbb{V}[\phi(w, P)]. \tag{2.4}
\]

Here, we recall that $\mathbb{V}[\phi(w, P)] = \mathbb{E}[\phi(w, P)^2] - \mathbb{E}[\phi(w, P)]^2$.

The optimal placement problem consists of finding the allocation vector $w$ that minimizes the profit variance. This problem is stated as:
\[
\min_w \Sigma_{\phi}(w). \tag{2.5}
\]

We assume that the optimal allocation vector (denoted as $w^*_1$) satisfies the constraint $\|w^*_1\|_2 = 1$ (it is a vector of unit length), where $\| \cdot \|_2$ denotes the Euclidean norm. This constraint is interpreted as the distribution of a finite amount of power among the network nodes. We note that the placement problem is scale-invariant. In other words, replacing $w \rightarrow \gamma w$ for some $\gamma > 0$ in the optimization
The eigenvectors can also be used to form a matrix loading allocation value and corresponding eigenvector we add the linear orthogonality constraint price matrix to obtain all the principal components. For instance, to obtain the second smallest eigenvalue we impose a constraint of the form \( \|w\|_2 = 1/\gamma \) will yield the same optimal allocation obtained with the constraint \( \|w\|_2 = 1 \).

A key observation is that the profit variance is related to the price covariance as \( \mathbb{V}[\varphi(w)] = w^T \mathbb{E}[(P - \mathbb{E}[P])(P - \mathbb{E}[P])^T] w \). This can be observed from the following series of implications:

\[
\mathbb{V}[\varphi(w)] = \mathbb{E}[(\varphi(w, P))^2] - \mathbb{E}[\varphi(w, P)]^2 = \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{N}} w_j w_k \mathbb{E}[P_j P_k] - \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{N}} w_j w_k \mathbb{E}[P_j] \mathbb{E}[P_k] = \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{N}} w_j w_k \text{Cov}(P_j, P_k) = w^T \mathbb{E}[(P - \mathbb{E}[P])(P - \mathbb{E}[P])^T] w. \tag{2.6}
\]

One can derive a similar relationship between the sample profit and covariance matrix to establish \( \Sigma_{\varphi}(w) = w^T \Sigma w \). Consequently, the optimal placement problem (2.5) can also be written as:

\[
\min_w \ w^T \Sigma w \text{ s.t. } \|w\|_2 = 1. \tag{2.7}
\]

This reveals that the placement problem is an eigenvalue problem. Accordingly, the optimal allocation vector \( w^*_1 \) is the eigenvector corresponding to the minimum eigenvalue \( \lambda^*_1 \) of the price covariance matrix \( \Sigma \). Moreover, the minimum eigenvalue is the minimum profit variance \( (\lambda^*_1 = \Sigma_{\varphi}(w^*_1)) \). We also note that the eigenvalue problem is a quadratic program (QP).

The eigenvalue problem is the basis of principal component analysis (PCA). The first principal component is given by \( (w^*_1)^T p_i \), \( i \in \mathcal{N} \). In PCA, one extracts the entire eigenvalue spectrum of the price matrix to obtain all the principal components. For instance, to obtain the second smallest eigenvalue and corresponding eigenvector we add the linear orthogonality constraint \( w^T w^*_1 = 0 \) to the eigenvalue problem (2.7). The solution of the new problem yields the eigenvector \( w^*_2 \) and corresponding eigenvalue \( \lambda^*_2 = \Sigma_{\varphi}(w^*_2) \). Since adding the orthogonality constraint restricts the feasible space, we have that \( \Sigma_{\varphi}(w^*_2) \geq \Sigma_{\varphi}(w^*_j) \). This procedure is repeated to obtain the entire set of eigenpairs \( w^*_j, \lambda^*_j, j \in \mathcal{N} \), where \( \lambda^*_n = \Sigma_{\varphi}(w^*_n) \) is the maximum possible cost variance (obtained with the loading allocation \( w^*_n \)). In our context, this procedure provides useful information because it allows us to obtain a family of allocations \( w^*_j, j \in \mathcal{N} \) and to rank them according to their profit variance. The eigenvectors can also be used to form a matrix \( W \) that can be used to project any price realization \( p_i \) into the space of the principal components as \( W p_i \). The projection can be used to identify clusters and/or outliers in the price data by analyzing only a subset of principal components.

### 2.2 Constrained Formulation

While mitigating profit variance is an important investment goal, obtaining a maximum expected profit is also important. Moreover, one often has constraints on the nature and capacity of assets that can be installed (e.g., generation, consumption). We thus extended the placement problem to capture these features. We impose a \( \ell_1 \)-norm constraint on the allocation vector \( w \) so that the total amount of power allocated adds up to one MWh and we add a condition that only one type of asset is allowed
to be built at one location (either generation or load). Consequently, we can decompose the node allocation $w_j$ into a generation $0 \leq w_{j,l}$ and a load component $-1 \leq w_{j,g} \leq 0$ (which are mutually exclusive). This gives the following multi-objective optimization problem:

$$\max_w \{ \mu_\varphi(w), -\Sigma_\varphi(w) \}$$  \hspace{1cm} (2.8a)

s.t. $\sum_{j \in N} (|w_{j,l}| + |w_{j,g}|) = 1$  \hspace{1cm} (2.8b)

$s.t. 0 \leq w_{j,g} \leq z_{j,g}, j \in N$  \hspace{1cm} (2.8c)

$-z_{j,l} \leq w_{j,l} \leq 0, j \in N$  \hspace{1cm} (2.8d)

$z_{j,l} + z_{j,g} \leq 1, j \in N$  \hspace{1cm} (2.8e)

$z_{j,l}, z_{j,g} \in \{0,1\}, j \in N$.  \hspace{1cm} (2.8f)

where $z_{j,l}$ and $z_{j,g}$ are binary variables that indicate if either a load or generation asset is installed at a particular location $j$. The constraint $z_{j,l} + z_{j,g} \leq 1$ indicates that either a load or a generator (but not both) can be installed at one location. Consequently, we have that $\sum_{j \in N} (|w_{j,l}| + |w_{j,g}|)$ is the total capacity of load and generators, and the constraint on the binary variables ensures that only one type of technology is allowed at each location. We can use the above formulation to characterize placement problems.

Unfortunately, the constrained placement problem is a mixed-integer QP. This problem is intractable for the data sets considered in this work. Motivated by this limitation, we consider the mean absolute deviation as a risk measure. This is given by:

$$MD(w) = \frac{1}{m} \sum_{i \in M} |\varphi(w, p_i) - \mu_\varphi(w)| \approx E[|\varphi(w, P) - \mu_\varphi(w)|].$$  \hspace{1cm} (2.9)

This risk measure is used to formulate the placement problem:

$$\max_w \{ \mu_\varphi(w), -MD(w) \}$$  \hspace{1cm} (2.10a)

s.t. $\sum_{j \in N} (|w_{j,l}| + |w_{j,g}|) = 1$  \hspace{1cm} (2.10b)

$s.t. 0 \leq w_{j,g} \leq z_{j,g}, j \in N$  \hspace{1cm} (2.10c)

$-z_{j,l} \leq w_{j,l} \leq 0, j \in N$  \hspace{1cm} (2.10d)

$z_{j,l} + z_{j,g} \leq 1, j \in N$  \hspace{1cm} (2.10e)

$z_{j,l}, z_{j,g} \in \{0,1\}, j \in N$.  \hspace{1cm} (2.10f)

which can be cast as a mixed-integer linear program and is thus easier to solve. The constrained placement problem is also scale-invariant. In other words, replacing $w \rightarrow \gamma w$ for some $\gamma > 0$ yields the same optimal allocations. This is because $MD(\gamma w) = \gamma MD(w)$, and $\mu_\varphi(\gamma w) = \gamma \mu_\varphi(w)$ (resulting in a linear scaling of the objective function). Consequently, imposing a constraint of the form $\|w\|_1 = 1/\gamma$ will yield the same allocation obtained with the unit-length constraint $\|w\|_1 = 1$. The constraints set capacity of load and generators, and the constraint on the binary variables ensures that only one type of technology is allowed at each location. We can use the above formulation to
understand the impacts of installing only certain types of assets or at certain locations. For instance, if we only wish to install generation assets, we set $z_{j,l}$ to zero.

3 Results and Discussion

In this section we use conduct a basic statistical analysis for an electricity market data set of the California ISO (CAISO) and use the optimal placement formulation to analyze economic incentives created by the DAM and RTM.

3.1 Volatility Analysis of Electricity Markets

In the DAM, electricity prices are updated hourly and market participants commit to buy or sell power one day before real-time operation, thus avoiding price volatility. This market produces one financial settlement per day. In the RTM market, prices are updated every 5 minutes and participants commit to buy or sell electricity over the course of the operating day. This market seeks to balance discrepancies between the day-ahead commitments and the actual real-time generation and loads seen in the power grid (e.g., due to unexpected variations in renewable power supply, equipment failures, and so on). The DAM and RTM work together to produce a multi-settlement system that balances power at different timescales and at thousands of network locations [4]. Usually, electricity prices in the DAM are usually less volatile but are on average higher than RTM prices, and thus market participants can participate strategically in either or both of these markets.

![Figure 2: Temporal average price (at different spatial locations) for CAISO in day-ahead (left) and real-time (right) markets.](http://zavalab.engr.wisc.edu)
profiles for the year at 2,234 different network locations. The data set contains over 19,569,840 price points for the DAM (one-hour time resolution) and 234,838,080 price points for the RTM (5-minute time resolution). We use this data to construct a space-time covariance matrix $\Sigma$ for both the DAM and RTM.

The results are visualized in Figures 2, 3, and 4. Figure 2 illustrates that the time-average price for both markets is in the range of 27-50 USD/MWh. The space-time average RTM price is 32.71 USD/MWh, which is 2.62% lower than the corresponding average DAM price of 33.59 USD/MWh. The differences illustrate that there is a premium in the DAM. Spatial patterns for both markets are quite similar, indicating that prices are dictated by the network topology. Figure 3 demonstrates temporal price volatility (standard deviation) at all locations. The temporal volatility in the DAM is consistently under 10 USD/MWh in most locations while the volatility in the RTM is in the range of
60-70 USD/MWh and reaches levels of 90 USD/MWh in some locations. The spatial average of the temporal volatilities was found to be 62.41 USD/MWh for the RTM, almost four times larger than in the DAM, which was only 12.93 USD/MWh. These results clearly indicate that RTM possesses greater temporal volatility. Figure 4 presents spatial volatility through time. We see that the DAM shows low spatial volatility (except in a few instances in the summer months) while the RTM shows more frequent spikes in spatial volatility. Based on this analysis we conclude that the RTM is more volatile than the DAM in both time and space. We also found that the temporal average of the spatial volatility was found to be 8.85 USD/MWh for the RTM and 5.60 USD/MWh for the DAM. We can thus see that, on average, spatial volatility is less significant than temporal volatility (which are 12.93 USD/MWh for DAM and 62.42 USD/MWh for the RTM).

We also computed the spatial correlation matrix based on the covariance matrix. Our results show that in the DAM, the average correlation is 0.67, that 99% of the total number of locations are positively correlated, and that the minimum correlation is -0.22. In the RTM, the average correlation is 0.82 and the minimum correlation is 0.00083. We conclude that a strong positive correlation exists in both electricity markets (prices at different locations tend to move in the same direction). This indicates that there is tight physical network coupling. As we will see next, strong positive correlation indicates that it is impossible to eliminate investment risk by simply investing in either generation or loads (a combination is needed). This would not be the case if we had strong negative correlation in the market.

3.2 Eigenvalue Analysis of Space-Time Covariance Matrix

Solving the basic placement formulation is equivalent to solving an eigenvalue problem. In Table 1 and Figure 5, we summarize the eigenvalue spectrum for both the DAM and RTM price covariance matrices (recall that the eigenvalues are the variances of the profit). Recall that both the DAM and RTM matrices have a total of 2,234 eigenvalues. The first 1,180 eigenvalues of the DAM price covariance are close to zero. For the RTM, the first 1,454 eigenvalues are close to zero (below a threshold value of $O(10^{-2})$). This indicates that many eigenvectors (allocations) give zero variance, meaning that many combinations of asset locations (given by the corresponding eigenvectors) can eliminate profit variance. An optimal strategy to eliminate risk is to place combinations of loads and generators at neighboring nodes (those with similar temporal price profiles). This can be visualized in Figure 6, where we show the optimal placement of assets (the eigenvectors) corresponding to the minimum eigenvalues. As can be seen, allocations of generation and load always appear in pairs next to each other and are of equal magnitude.

The largest eigenvalue (the maximum possible profit variance) is $O(10^5)$ for the DAM and $O(10^6)$ for the RTM, indicating that there is more volatility in the RTM (reinforcing the observations made with basic statistical analysis). In Figure 7, we present the optimal allocations corresponding to the maximum variance. The strategy here is to place the same asset type (in this case power generation) at all nodes. The maps also reveal areas the are strongly positively correlated (so the strategy to maximize variance is to allocate more generation at such locations). Obviously, this strategy is not optimal from an investment standpoint but highlights some interesting properties of the behavior of
Table 1: Eigenvalues for DAM and RTM covariance matrices.

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>DAM</th>
<th>RTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>$-4.25 \times 10^{-12}$</td>
<td>$-4.59 \times 10^{-11}$</td>
</tr>
<tr>
<td>$\lambda_{10}$</td>
<td>$-2.09 \times 10^{-14}$</td>
<td>$-1.90 \times 10^{-12}$</td>
</tr>
<tr>
<td>$\lambda_{100}$</td>
<td>$-2.86 \times 10^{-16}$</td>
<td>$-5.74 \times 10^{-16}$</td>
</tr>
<tr>
<td>$\lambda_{500}$</td>
<td>$2.91 \times 10^{-18}$</td>
<td>$5.80 \times 10^{-17}$</td>
</tr>
<tr>
<td>$\lambda_{1000}$</td>
<td>$5.78 \times 10^{-4}$</td>
<td>$4.68 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\lambda_{1500}$</td>
<td>$0.24$</td>
<td>$0.016$</td>
</tr>
<tr>
<td>$\lambda_{2000}$</td>
<td>$5.87$</td>
<td>$9.20$</td>
</tr>
<tr>
<td>$\lambda_{2100}$</td>
<td>$18.35$</td>
<td>$74.15$</td>
</tr>
<tr>
<td>$\lambda_{2200}$</td>
<td>$300.62$</td>
<td>$2806.04$</td>
</tr>
<tr>
<td>$\lambda_{2234}$</td>
<td>$2.54 \times 10^5$</td>
<td>$7.31 \times 10^6$</td>
</tr>
</tbody>
</table>

Figure 5: Cumulative eigenvalue spectrum for the DAM (left) and the RTM (right) covariances.

3.3 Risk vs. Mean Profit Trade-off for the DAM

We used the optimal placement formulation to analyze trade-offs between risk and expected profit. In Table 2 and Figure 8 we present the trade-off for the DAM. We can make a number of interesting observations. First, it is clear that to maximize expected profit it is optimal to centralize facilities (these facilities are simply installed at locations with large mean price). In this case, obviously, the type of asset to install is generation and the expected profit is 52.76 $/MWh. This strategy, however, results in a large risk (an MD value of 24.03 $/MWh). We can also see that the mean deviation is significant, representing half of the expected profit, which is due to the high temporal volatility of the prices. The trade-off trends also indicate that installation of a larger number of smaller power generators (diversifying generation among multiple locations) can substantially decrease risk. For instance, by increasing the number of generators to five, we see that the risk is decreased by 50%
and this only decreases the expected profit by 15%. This illustrates that there is a strong nonlinear trade-off between expected profit and risk.

From Table 2 and Figure 8 we see that further reductions in risk require the installation of both generation and loads. In particular, elimination of risk cannot be achieved through the use of either just generation or just loads (due to the positive correlation of prices). In the hypothetical case in which market prices were negatively correlated, installing the same asset type would be sufficient to fully mitigate risk. Consequently, the limiting value of risk for single asset type is an indicator of the degree of positive correlation in the market. Figure 9 shows optimal placement locations for low-risk and high-risk cases. We see that high-risk is achieved by placing only generation assets while
low-risk is achieved by diversifying loads and generation.

### Table 2: Risk vs. expected profit trade-off for DAM.

<table>
<thead>
<tr>
<th>Risk ($/MWh)</th>
<th>Expected Profit ($/MWh)</th>
<th># of Loads</th>
<th># of Generators</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.03</td>
<td>52.76</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>18.15</td>
<td>49.15</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>12.19</td>
<td>44.22</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>8.31</td>
<td>39.24</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>5.75</td>
<td>34.16</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>5.06</td>
<td>31.85</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>4.25</td>
<td>28.15</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>3.15</td>
<td>22.13</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>1.33</td>
<td>10.63</td>
<td>29</td>
<td>12</td>
</tr>
<tr>
<td>0.16</td>
<td>1.92</td>
<td>142</td>
<td>112</td>
</tr>
<tr>
<td>0.055</td>
<td>0.80</td>
<td>287</td>
<td>254</td>
</tr>
</tbody>
</table>

An interesting trade-off point that we see in Table 2 is that in which we obtain a risk of $MD = 5.06$ USD/MWh and expected profit of $\mu_\varphi = 31.85$ USD/MWh (this is the solution for minimum possible risk achieved with only generation assets). In this solution seven generation locations achieve an mean absolute deviation of 5.06 MWh and a expected profit of 31.85 MW (78.94% of the risk is reduced while 39.63% of the profit is sacrificed). In Table 3 we show the power allocation to each of the seven locations. We see that two locations share 90% of the total generation (these seek to maximize expected profit) while 10% of the generation is split in small generators (these seek to minimize risk). From Table 2 we see that the use of just two generators incurs a large risk. Consequently, investing in smaller generators is key to mitigate risk. From these results we also conclude that further diversification of generation does not provide significant benefits in risk mitigation.
Table 3: Optimal allocation for case with $MD = 5.06$ USD/MWh and $-\mu_\varphi = 31.85$ USD/MW in the DAM.

<table>
<thead>
<tr>
<th>Location</th>
<th>$w_i$ (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEORBLF_7_B1</td>
<td>0.60</td>
</tr>
<tr>
<td>JBBLACK1_7_B1</td>
<td>0.31</td>
</tr>
<tr>
<td>DELNORTE_LNODED50</td>
<td>0.037</td>
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<tr>
<td>HMBUNIT2_7_GN010</td>
<td>0.023</td>
</tr>
<tr>
<td>HMBLTBY_6_N003</td>
<td>0.017</td>
</tr>
<tr>
<td>TOPAZC1_7_N021</td>
<td>0.010</td>
</tr>
<tr>
<td>BAFCOG12_7_B1</td>
<td>0.00090</td>
</tr>
</tbody>
</table>

3.4 Risk vs. Expected Profit Trade-off for the RTM

Trade-off analysis for the RTM was performed by using price data with a time resolution of 20 minutes. The reason is that the placement problem is intractable at higher resolutions. The results are summarized in Table 4 and Figure 10. We can observe similar trends to those found in the DAM. In contrast with the DAM results, the risk for RTM is higher (which is consistent with the results obtained using the eigenvalue analysis). Compared to the DAM, more diversification of generation is needed to decrease the risk by the same amount (due to the higher volatility in RTM). We also observe that a combination of loads and generators is needed to fully eliminate risk and that the expected profit obtained with the RTM and DAM are similar.

Figure 11 shows high-risk and low-risk allocations. High-risk allocations with large expected profit favor centralization of assets while low-risk ones favor decentralization of assets. Moreover, this indicates that assets capable of providing simultaneous provision of generation and load (e.g.,
microgrids or batteries) can be used to mitigate risk. Our analysis also indicates that electricity markets provide significant incentives to modularize power-intensive assets (e.g., manufacturing facilities and data centers). For instance, decentralization of ammonia systems can help mitigate risk associated with the high consumption of electricity in refrigeration systems.

The risk estimated with the 20-minute formulation underestimates that of the 5-minute counterpart. We can see, however, that the 20-minute resolution data already reveals that much higher risk is observed in RTM relative to DAM. This observation is also confirmed using the eigenvalue analysis (which was performed using the 5-minute resolution data). Moreover, we expect similar trade-off trends by using higher time resolutions.

Table 4: Risk vs. expected profit trade-off for the RTM.

<table>
<thead>
<tr>
<th>Risk (USD/MWh)</th>
<th>Expected Profit (USD/MWh)</th>
<th># of Loads</th>
<th># of Generators</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.85</td>
<td>51.59</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>29</td>
<td>47.03</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>43.75</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>19</td>
<td>40.22</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>37.70</td>
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<td>6</td>
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<tr>
<td>13</td>
<td>33.98</td>
<td>0</td>
<td>13</td>
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</tr>
<tr>
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<td>0</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>19.71</td>
<td>6</td>
<td>18</td>
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<tr>
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<td>13.44</td>
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<td>22</td>
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<td>45</td>
<td>47</td>
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<tr>
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<tr>
<td>0.2</td>
<td>0.76</td>
<td>130</td>
<td>157</td>
</tr>
</tbody>
</table>

3.5  Computational Considerations

The basic placement problem is an eigenvalue problem that can be readily solved for both the DAM and the RTM data (even at 5 minute resolutions). The constrained placement formulation (2.10), on the other hand, is a large-scale mixed-integer linear program. The RTM problem (with 20-min resolution) contains 85,544 constraints and 61,496 variables (4,468 binary) while the DAM problem contains 32,985 constraints, 21,989 continuous variable (4,468 binary). The problems where solved with Gurobi and solution times range from 1.5 hours to 5 hours. The long times are due to significant symmetries in the problem (i.e., many allocation combinations achieve the same optimal objective).
This degeneracy was revealed by the eigenvalue analysis (which indicates that the price covariance has a large number of zero eigenvalues). The RTM problem is intractable with time resolutions below 20 minutes. We are currently investigating strategies to decompose the placement problem in order to be able to scale to higher time resolutions. In particular, this problem has the interesting property that it only has a single coupling constraint. Consequently, one can develop specialized Lagrangian decomposition schemes that achieve high parallel execution efficiency.

4 Conclusions and Future Work

This paper examines economic incentives created by space-time dynamics of day-ahead and real-time electricity markets. We developed an optimal technology placement formulation that seeks to identify optimal strategies to maximize expected profit and minimize risk. We have shown that a pure risk minimization formulation can be cast as an eigenvalue problem for the price covariance...
matrix. We also developed more sophisticated formulations that capture different technology asset types (e.g., generation or loads) and risk measures using mixed-integer programming techniques. Our analysis for the CAISO market reveals that significantly more temporal (as opposed to spatial) volatility is observed in both the DAM and RTM markets. The RTM also has more volatility in general. Our analysis also reveals that both markets exhibit positive spatial correlation in prices, indicating that it is impossible to fully eliminate risk by using only either generators or loads. Consequently, decentralizing technologies of the same type has significant but limited impacts on risk mitigation. Full risk mitigation can only be achieved by combinations of generation and load assets (which can be achieved with microgrids or batteries). Our analysis also indicates that electricity markets provide significant incentives to modularize power-intensive technologies (e.g., manufacturing and data centers). This is of particular relevance due to recent interest in the deployment of small-scale modular technologies.

Our analysis assumes no temporal correlation in prices, which is a conservative assumption. Capturing simultaneous spatial and temporal correlations requires the use of more sophisticated techniques. Specifically, we are interested in investigating recently-developed dynamic principal component analysis techniques to conduct space-time analysis of market data. It is necessary to extend such techniques to capture constraints and that enable temporal flexibility; our current formulation assumes that technologies provide a fixed capacity all the time. We are also interested in investigating decomposition strategies to handle high temporal resolutions for RTM data.

Acknowledgments

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References


