Stochastic Optimization Formulations for Long-Term Extreme Load Mitigation in Wind Turbines

Yankai Cao, Fernando D’Amato, and Victor M. Zavala

Abstract—We propose stochastic optimization formulations to enforce extreme mechanical load requirements in controller design procedures. The formulations use a probabilistic constraint that captures the long-term probability of exceeding an extreme load threshold (as described by the IEC-61400). We use the observation that extreme loads follow a generalized extreme value distribution to obtain an explicit algebraic characterization of the probabilistic constraint. We illustrate how to use the formulations to find design parameters for pitch and torque controllers that maximize power output while maintaining the original extreme load characteristics. We also use the formulation to explore the performance of a hypothetical MPC controller. The proposed formulations are cast as large-scale nonlinear programs, and can be solved in less than 1.5 hours using a state-of-the-art parallel interior-point solver.

Index Terms—Stochastic, optimization, fatigue, wind turbines.

I. INTRODUCTION

INDUSTRIAL wind turbines are designed to operate through a lifetime of more than 20 years and under highly uncertain weather conditions. Strong wind conditions can compromise the integrity of the turbine if they are not properly handled through the control system. To prevent structural damage and high life consumption rates, the International Electrotechnical Commission (IEC) standards require wind turbine designers to ensure that the turbine and associated control system does not exceed critical load conditions when subjected to a multiplicity of operating scenarios. Many of these critical load conditions are derived using statistical load extrapolation methods [1], [2], [3]. Such procedures seek to use limited short-term wind and load data to assess the long-term probability of exceeding a certain load threshold. Extrapolation techniques are based on a powerful result from statistics known as the extreme value theorem that states that the maximum of a sequence of independent random variables follows a generalized extreme value distribution. The existence of such a distribution enables the estimation of probabilities of rare long-term events.

Control of industrial wind turbines requires a careful trade-off between energy capture and compliance with structural loads. Control strategies for industrial wind turbines are typically based on control architectures that regulate power and rotor speed by operating generator torque and collective blade pitch angle. In addition, supervisory control systems are used to mitigate loads and perform shut-down procedures [4], [5]. As a result, designing a viable control architecture involves extensive and time-consuming simulations with different controller settings that satisfy IEC requirements.

Recent research activity in wind turbine control [6], [7], [8], [9], [10] has focused on Model Predictive Control (MPC) technology. Such strategies have been used to regulate power and speed and to enforce fatigue and load constraints. MPC is a powerful optimization-based technology that can aid standard controllers, as it can directly accommodate detailed turbine models and constraints of different forms and with this anticipate wind events and capture multivariable interactions. In particular, MPC strategies can perform simultaneous blade pitch and generator torque control while maximizing power and mitigating extreme loads. For instance, in the work of [6], it is shown that under an extreme gust event, an MPC strategy can reduce the tower base moment by up to 15% compared to standard controllers. Similarly, the work of [7] uses a nonlinear MPC formulation to demonstrate that some loads can be reduced by up to 50% under extreme gusts without negative impact on overall power production. The nonlinear MPC formulation of [10] also shows that it can reduce the tower bending moment by up to 40%. The work in [11] uses multi-objective optimization to explore the trade-offs between generated power and structural loads. Simplified MPC strategies have also been reported in the literature that seek to overcome computational complexity by using linearized model representations [8], [9].

A limitation of MPC strategies reported in the literature is that they analyze robustness to diverse wind scenarios on a case by case basis and do not capture long-term extreme load constraints as required by IEC standards. Unfortunately, computational procedures used in long-term statistical extrapolation are complex and not trivial to implement in controller design and MPC formulations. Because of this, the effect of control strategies on wind turbine loads is often performed a posteriori and not a priori (by design).

In this work, we propose stochastic optimization formulations that enforce extreme load constraints as required by IEC standards directly in controller design procedures. The formulations exploit the fact that the cumulative distribution function (CDF) associated to the extreme value distribution has an explicit algebraic representation. Moreover, the CDF can be easily fit to actual data using moment matching. We demonstrate the benefits of the proposed approach by determining optimal parameters for pitch and torque controllers and by evaluating a hypothetical MPC supervisory strategy. Our results use a simplified wind turbine model and control architectures, which allow us to perform validation against an exhaustive search procedure. We show that the optimization formulations, which are cast as large-scale nonlinear programs

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with up to 7.5 million variables, can be solved in less than 1.3 hours by using a state-of-the-art parallel solver.

II. OPTIMIZATION FORMULATIONS

In this section, we present deterministic and stochastic variants for a controller design problem. The goal of the deterministic formulation is to compute optimal controller parameters that impose constraints on the maximum load experienced by the turbine under a given known wind speed profile and that maximize extracted power. In the stochastic optimization formulation, the goal is to compute optimal controller parameters that maximize expected power over an uncertain wind power profile (where uncertainty is captured in the form of scenarios). The formulation also imposes probabilistic constraints for the exceedance of given load thresholds in the long-term loads, as required by the IEC61400-1 standard. Appendix A describes all variables, parameters, and units of the model.

A. Wind Turbine Model

We motivate our work by using a wind turbine model described by the following system of differential and algebraic equations (DAEs) [10]:

\[ w_r(t) = \frac{1}{J} (M_z(t) - N_g T_{gen}(t)) \]  
(II.1a)

\[ \dot{x}(t) = v_x(t) \]  
(II.1b)

\[ \dot{v}_x(t) = \frac{-1}{m_{Tc}} (-c_{Tc} v_x(t) - k_{Tc} x(t) + F_z(t)) \]  
(II.1c)

\[ V_{rel}(t) = V(t) - v_x(t) \]  
(II.1d)

\[ \lambda(t) = w_c(t) R_c / V_{rel}(t) \]  
(II.1e)

\[ C_t(t) = ThrustCoeff(\theta(t), \lambda(t)) \]  
(II.1f)

\[ C_m(t) = TorqueCoeff(\theta(t), \lambda(t)) \]  
(II.1g)

\[ F_z(t) = \frac{1}{2} \rho V_{rel}(t)^2 A_C(t) \]  
(II.1h)

\[ M_z(t) = \frac{1}{2} \rho V_{rel}(t)^2 A_R C_m(t) / \lambda(t) \]  
(II.1i)

\[ y_p(t) = T_{gen}(t) w_g(t) (1 - P_i) \]  
(II.1j)

\[ y_L(t) = H(k_{Tc} x(t) + c_{Tc} v_x(t)) \]  
(II.1k)

Equations (II.1f) and (II.1g) are highly nonlinear relationships and are given in [10]. The wind turbine is equipped with single-loop controllers for torque \( T_{gen}(t) \) and pitch angle \( \theta(t) \). The generator torque controller used has the form proposed in [12]. Given a set-point for rated power \( P_{rated} \) and the generator speed \( w_g(t) = w_r(t) N_g \), the controller adjusts the torque according to:

\[
T_{gen}(t) = \begin{cases} 
\kappa w_g(t)^2, & \text{if } w_g(t) \leq w_{g2} \\
\kappa w_{g2}^2 + \frac{w_g(t)-w_{g2}}{w_{g25}-w_{g2}} (P_{rated}/w_{g25} - \kappa w_{g2}^2), & \text{if } w_{g2} < w_g(t) \leq w_{g25} \\
P_{rated}/w_{g25}, & \text{if } w_g(t) > w_{g25}
\end{cases}
\]

(II.2)

where \( \kappa > 0 \) is a parameter. As shown in Fig. 1, the control law is piecewise linear (non-smooth) and has three regions of interest. The parameter \( w_{g2} \) denotes the threshold between region 2 and region 2.5 and \( w_{g25} \) denotes the threshold between region 2.5 and region 3. This strategy defines the torque setpoints in region 3 to provide rated power. For the blade pitch controller, we use a proportional-integral control law [12] that adjusts the pitch angle given a set-point for the generator speed \( w_{g_{ref}} \):

\[
\Delta w_g(t) = w_g(t) - w_{g_{ref}}
\]

(II.3a)

\[
w_{g_{int}}(t) = \int_0^t \Delta w_g(\tau) d\tau
\]

(II.3b)

\[
w_{g_{int}}(t) = \max\{w_{g_{int}}(t-1) + \Delta w_g(t) h, 0\}
\]

(II.3c)

\[
\theta(t) = \max\{K_p \Delta w_g(t) + K_i w_{g_{int}}(t), 0\}
\]

(II.3d)

The work in [12] provides a recommendation for the value of \( K_p \) and \( K_i \) but these parameters depend on the value \( \theta \). We simplified these values by fixing the value of \( K_p \) and \( K_i \) at the maximum value of \( \theta \). In particular, we use the parameters \( K_p = 0.216 \) and \( K_i = 0.0924 \). By using the control laws described before, the optimization problem has four degrees of freedom, given by the pitch and torque controller parameters (set-points) \( u = (P_{rated}, w_{g_{ref}}, w_{g2}, w_{g25}) \). The parameters \( P_{rated} \), \( w_{g2} \), and \( w_{g25} \) determine the performance of the local controller for torque, and \( w_{g_{ref}} \) determines the performance of the pitch controller. In this work we use a modified torque control law, which allows power to go above rated values during short time intervals:

\[
T_{gen}(t) = \begin{cases} 
\kappa w_g(t)^2, & \text{if } w_g(t) \leq w_{g2} \\
\kappa w_{g2}^2 + \frac{w_g(t)-w_{g2}}{w_{g25}-w_{g2}} (P_{rated}/w_{g25} - \kappa w_{g2}^2), & \text{if } w_{g2} < w_g(t) \leq w_{g25} \\
P_{rated}/w_{g25}, & \text{if } w_g(t) > w_{g25}
\end{cases}
\]

(II.4)

Here, instead of maintaining constant power in region 3, we adopt the strategy proposed in [10] to maintain the rated torque value whenever the rotor speed exceeds its rated value. We emphasize that the only difference between the modified and the standard control law is in region 3 while the performance in both region 2 and region 2.5 are the same, as shown in Figure 1. In particular, in region 3 of the modified formulation, the generator torque remains constant as \( w_g(t) \) increases, and thus the power output increases as \( w_g(t) \) increases. In Section IV we show that this modification increases performance and this helps us demonstrate the benefits of the proposed formulations.
We note that the controllers have only four design parameters to optimize; consequently, their optimal values can be found by exhaustive search. These parameters will be used to validate more advanced optimization formulations.

B. Deterministic Optimization Formulation

We represent the deterministic wind turbine optimization problem in the following abstract form:

\[
\max_{u \in U} \frac{1}{T} \int_{T} y_P(t) dt \quad \text{II.5a}
\]

\[
\text{s.t. } (y_P(t), y_L(t)) = \phi(u, V(t)), \quad t \in \mathcal{T} \quad \text{II.5b}
\]

\[
y_L(t) \leq \bar{y}_L, \quad t \in \mathcal{T} \quad \text{II.5c}
\]

where \( t \in \mathcal{T} := [0, T] \), \( y_P(t) \) is the wind turbine power, \( y_L(t) \) is the mechanical load with associated threshold \( \bar{y}_L \), and \( V(t) \) is the wind profile. The controls \( u \) (the controller parameters) are restricted to the closed set \( U \), which is a four-dimensional hypercube defined by the following constraints:

\[
P_{\text{rated0}} \leq P_{\text{rated}} \leq 1.2P_{\text{rated0}} \quad \text{II.6a}
\]

\[
w_{g_{\text{ref}}} \leq w_{g_{\text{v}}}, \quad w_{\text{g}_{\text{ref}}} \leq 1.2w_{g_{\text{ref}}} \quad \text{II.6b}
\]

\[
w_{g_{\text{v}}}, \quad w_{g_{\text{v}}} \leq w_{g_{\text{v}}} \leq 1.2w_{g_{\text{v}}} \quad \text{II.6c}
\]

\[
w_{g_{\text{v}}} \leq w_{g_{\text{v}}} \leq 1.2w_{g_{\text{v}}} \quad \text{II.6d}
\]

An important practical problem is that power maximization often conflicts with the mechanical load experienced by the turbine (i.e., the higher the power extracted the higher the load). Consequently, it is critical to carefully trade-off these metrics so as to prevent putting the turbine at mechanical risk.

In summary, the optimization formulation is a controller design formulation that seeks to identify optimal parameters for the low-level torque and pitch controllers that maximize extracted power and that mitigate extreme loads. Additionally, we consider the problem of allowing MPC to directly compute the control actions on pitch and torque actuation (by eliminating the control laws). This approach can provide valuable information about best achievable performance. We discuss this more advanced setting in Section IV.

C. Standard Stochastic Optimization Formulation

We use a stochastic formulation to account for wind variability (see Figure 3). We model the wind profile as a random variable \( V(t) \) and use \( V(t, \omega) \) to denote a realization \( \omega \in \Omega \) obtained from the probability density \( p(V(t)) \). Here, \( \Omega \) represents a scenario (or realization) set. We note that, because the wind profile propagates through the wind turbine model, the turbine extracted power and load outputs are also random variables. To maintain notational consistency, the random power output is denoted simply as \( y_P(t, \omega) \) with realizations \( y_P(t, \omega) \) and the random load is \( y_L(t, \omega) \) with realizations \( y_L(t, \omega) \). The posterior probability density of the power output and for the load are denoted as \( p_P(y_P(t) | V(t), u) \) and \( p_L(y_L(t) | V(t), u) \). Realizations from these posterior densities (denoted as \( y_P(t, \omega) \) and \( y_L(t, \omega) \)) are obtained by propagating the wind speed realizations \( V(t, \omega) \) and \( u \) through the model.

One can formulate a stochastic program to find the parameters \( u \) that maximize expected power \( \mathbb{E} \left[ \frac{1}{T} \int_{T} y_P(t, \omega) dt \right] \) while satisfying the probabilistic constraint:

\[
\mathbb{P} \{ y_L(t, \omega) \leq \bar{y}_L, \quad t \in \mathcal{T} \} \geq \alpha, \quad \text{II.7}
\]

where \( \alpha \in [0,1] \) is a probability level. The probabilistic constraint (II.7) can also be expressed as \( \mathbb{P} \{ y_L(t, \omega) \leq \bar{y}_L \} \geq \alpha \). This constraint enforces that the probability that the maximum load over the time horizon \( \mathcal{T} \) exceeds the threshold is no more than \( 1 - \alpha \).

The CDF of the extreme load \( \max_{t \in \mathcal{T}} y_L(t, \omega) \) is denoted as \( F_{\text{max}}(y_L(\alpha)) \) and \( y_L(\alpha) \) is the \( \alpha \)-critical value (fractile/quartile) of the distribution. We refer to this critical load value as the characteristic load. The probabilistic constraint can also be written as \( F_{\text{max}}(\bar{y}_L(\alpha)) \geq \alpha \) or one can use the constraint on the quartile \( y_L(\frac{1}{4}) \leq \bar{y}_L \). In particular, we note that \( y_L(\alpha) \leq \bar{y}_L \) implies \( \mathbb{P} \{ y_L(t, \omega) \leq \bar{y}_L \} \geq \alpha \).

The probabilistic load constraint (II.7) is enforced over the time domain \( \mathcal{T} \). Clearly, if rare and extreme load scenarios over horizons of years need to be captured, this approach would be computationally impractical. Moreover, the conditional density \( F_{\text{max}}(\cdot, \cdot) \) does not have a closed form due to the nonlinearity of the wind turbine model. We now describe a procedure to approximate this long-term probabilistic constraint by using statistical extrapolation methods.

D. Stochastic Formulation Based on Statistical Extrapolation

The IEC-61400 standard proposes a procedure to enforce long-term (1-year and 50-year) probabilistic constraints on extreme loads. This is done by using existing short-term load data to perform long-term extrapolation [1]. This method has been shown to provide accurate estimates of long-term extreme loads [2], [3]. The key idea behind load extrapolation is to define a sufficiently long horizon (that we also define as \( \mathcal{T} \) with some abuse of notation) under which the extreme load \( \max_{t \in \mathcal{T}} y_L(t, \omega) \) achieves statistical steady-state and with this ensure that it becomes statistically independent of the extreme load at a subsequent time horizon \( \mathcal{U} \). With this, we can split a long horizon of interest into a set of short time horizons and define the extreme load over the long horizon as the maximum of the extreme loads over the short horizons. The length of the short horizon is typically chosen to be 10 minutes [1]. Load extrapolation methods are based on a powerful result from statistics known as the extreme value theorem that states that the maximum of a set of independent random variables always distributes according to the generalized extreme value distribution. Here, we focus on the Gumbel distribution, which is a special case and which is suggested by the IEC-61400 standard. The extreme value theorem is an analog of the celebrated central limit theorem, which states that the sum of independent random variables is always normally distributed.

Once it is known that the extreme load distribution follows a certain algebraic form, we can find the parameters of the distribution that best fit the data (by using moment matching). This can be achieved by using the following peak-over-threshold procedure (described in the IEC-61400 standard):

- Each realization of the wind \( V(t, \omega) \), \( t \in \mathcal{T} \) has an associated time-average wind speed \( \bar{V} \). We discretize the time-average wind-speed range into \( j \in B := \{1, \ldots, B\} \) bins (the bins typically span the range [3, 25] m/s and we use \( B = 23 \) bins). We categorize every realization of the wind into a single bin \( j \) with corresponding mean wind speed \( V_j \) and create scenario sets \( \Omega_j \) with \( \Omega = \cup_{j \in B} \Omega_j \).
• For a given wind speed bin $V_j$, we gather wind turbine power realizations under fixed controller parameters $u$ and $N_j$ wind profiles from the scenario set $\Omega_j$. These realizations have associated loads $y_L(t,\omega)$, $\omega \in \Omega_j$ from which we extract a total of $n_j$ local load maxima that are contained in a vector $L_j \in \mathbb{R}^{n_j}$. The maxima are defined as all local load maxima above a given threshold $L_{\text{ex}}$.

• We fit the local maxima to a Gumbel distribution. The parameters of the distribution are $c_j = \sqrt{6} \sigma_j$ and $L_j^0 = L_{\text{ex}} - c_j \gamma$ for $j \in B$. Here, $\sigma_j$ is the standard deviation of $L_j$, $L_{\text{ex}}$ is the mean of $L_j$, and $\gamma$ is the Euler constant.

• We compute the long-term exceedance probability for the load $P_{\text{ex}}(y_L)$ by integrating over all operating wind speeds. This is done by using the equations $F_j(y_L) = \exp \left(-\exp \left(-\left((y_L - L_j^0)/c_j\right)\right)\right)$, $j \in B$ and $P_{\text{ex}}(y_L) = \sum_{j \in B} p_j (1 - F_j(y_L)^{n_j})$. Here, $F_j(\cdot)$ is cumulative Gumbel density (probability that the load is below the critical value $y_L$) conditional to the mean wind speed $V_j$ and $p_j$ is the probability that the mean wind speed is within the bin $j \in B$. The probabilities can be computed by assuming that the mean wind speed follows a Rayleigh distribution.

• We solve the equation $P_{\text{ex}}(y_L) = \alpha_{50\text{yr}}$ with $\alpha_{50\text{yr}} = 10^{-5} = 3.8 \times 10^{-7}$ for $y_L$ to obtain the 50-year characteristic load $\hat{y}_L(\alpha_{50\text{yr}})$ or solve $P_{\text{ex}}(y_L) = \alpha_{1\text{yr}}$ with $\alpha_{1\text{yr}} = 10^{-9} = 1.9 \times 10^{-5}$ for $y_L$ to obtain the 1-year load $\bar{y}_L(\alpha_{1\text{yr}})$.

The characteristic load $\hat{y}_L(\alpha)$ is the critical load (a fractile) that exceeds the threshold $\bar{y}_L$ with probability $\alpha$. The 50-year characteristic load is to be interpreted as the load that, on average, is exceeded only once every 50 years. Similarly, the 1-year load is to be interpreted as the load that, on average, is exceeded only once every one year.

In a simulation setting, we obtain wind turbine realizations of the loads $y_L(t,\omega)$ by performing dynamic turbine model simulations for each given realization of the wind $V(t,\omega)$. The IEC-61400 load extrapolation procedure can thus be represented by using an input-output function of the form $\hat{y}_L(\cdot,\cdot)$ where $\hat{y}_L(\cdot,\cdot)$ is the characteristic load at probability level $\alpha$ (e.g., 1-year, 20-year, 50-year). The load extrapolation procedure allows us to impose a constraint on the characteristic load of the form $\hat{y}_L(\alpha) \leq \bar{y}_L$. This constraint on the fractile $\hat{y}_L(\alpha)$ implies that the probability that the load exceeding the threshold $\bar{y}_L$ is less than $1 - \alpha$ (i.e., the probability that the load is below $\bar{y}_L$ is greater than $\alpha$).

### III. Solution of Stochastic Formulation

In this section we propose simulation-based and optimization-based (all-at-once) procedures to solve the stochastic optimization formulation.

#### A. Exhaustive Simulation-Based Search

The number of degrees of freedom $u$ in the stochastic formulation is small. Consequently, it is possible to perform an exhaustive search of the space $u \in \mathcal{U}$. This approach has the advantage that it allows us to explore the whole decision space and provides information on how power and maximum loads change with the controller parameters. Moreover, this approach is parallelizable and does not require derivative information. Consequently, with this approach we can handle non-smooth functions associated to the pitch and torque control laws and we can directly implement the peak-over-threshold procedure described in the IEC-61400 standard. On the other hand, even when parallel simulations are performed, an extremely large number of simulations will be needed to span the entire decision space and the method will not scale. Scalability issues will also become evident when more degrees of freedom need to be considered (as in MPC formulations or more complex control architectures). The simulation-based search procedure follows the basic steps:

1) Given $\alpha$, for each $u \in \mathcal{U}$, DO:
2) For each wind scenario $V(t,\omega)$, simulate the dynamic model equations and compute expected power $\mathbb{E}\left[\frac{1}{T} \int_T^T y_P(t,\omega)dt\right]$ and loads $y_L(t,\omega)$.
3) Use the IEC-61400 load extrapolation procedure to match observed extreme load moments to the moments of the Gumbel distribution.
4) Use moments to compute characteristic load $\hat{y}_L(\alpha)$.

After exploration of the entire control space $u \in \mathcal{U}$, determine the combination that maximizes expected power and satisfies the characteristic load constraint $\hat{y}_L(\alpha) \leq \bar{y}_L$.

#### B. Optimization-Based Search

In an all-at-once optimization approach, we discretize the wind turbine model using a Radau collocation scheme to obtain a purely algebraic representation of the wind turbine model and with this we bypass the need to simulate the model repetitively [13]. Another benefit of this approach is that we can obtain derivative information and handle a wider range of degrees of freedom and constraints (e.g., to consider different parameters in which an MPC controller overrides low-level controllers). The presence of non-smooth pitch and torque controller laws, however, requires of suitable smooth reformulations. Moreover, the load IEC-61400 extrapolation procedure requires an automated procedure to extract local maxima from load profiles to fit the long-term extreme value distribution. This is inherently a dual control problem that seeks to maximize power while maximizing information extraction to perform long-term load inference. We now describe strategies to address these issues.

To ensure differentiability, the min and max functions used in the control laws are approximated using sigmoidal smooth functions. In particular, a function of the form $\max\{x, y\}$ is approximated using the function $x + \frac{1}{2} \log(1 + e^{-\epsilon(x-y)})$ while a function of the form $\min\{y, x\}$ can be approximated using $x - \frac{1}{2} \log(1 + e^{-\epsilon(y-x)})$. Here, $\epsilon > 0$ is a smoothing parameter that controls the quality of the approximation.
We extract a total of $\Omega$ subsets we denote the maximum load in stage $\omega$ before. Here, we partition the horizon in approach instead of the peak-over-threshold block maximum optimization setting, we use the so-called $\eta$ min $u$ s.t. $(\omega \in T/S)$ length sets $T$ and extract the maximum load in each stage. The characteristic load obtained with the block maximum method is close to that of the peak-over-threshold method.

Formulation (III.8) implements the proposed block-maximum load extrapolation procedure. We treat this problem as a bi-objective problem that seeks to extract the maximum power while extracting maximum peak load information to perform long-term load extrapolation. We highlight that these objectives are conflicting because maximum power occurs at the maximum load. Because of this, the expected power is handled through the constraint (III.8m), where $\bar{P}$ is a minimum power level. The probability of exceedance $P_{\text{ex}}(y_L)$ is close to zero and this introduces numerical difficulties. To deal with this, we approximate $P_{\text{ex}}(y_L)$ using the Taylor expansion $P_{\text{ex}}(y_L) \approx \sum_{j \in B} p_j (1 - F_j (y_L))$, which is much easier to handle computationally. We highlight that the proposed formulation is primarily motivated by the need to enforce probabilistic load constraints using statistical extrapolation. Consequently, the choice of the economic function to be optimized is not relevant.

IV. CASE STUDIES AND RESULTS

In this section we demonstrate the benefits of the proposed stochastic optimization formulations. We compare performance using nominal and optimal controller parameters obtained with a simulation-based and an optimization-based search. The parallel simulation setting is implemented in Julia and the optimization setting is implemented in Plasmo.jl, which is a Julia-based modeling framework that facilitates the construction and analysis of structured optimization models [14]. The structure of the problems is exploited using PIPS-NLP, which is an interior point solver that uses parallel Schur decomposition techniques to perform linear algebra [15]. Due to constraints on space, we only present a summary of the most relevant numerical results. A detailed version of this paper is available at http://zavalab.engr.wisc.edu.

A. Exhaustive Simulation-Based Search

We first show the results of the simulation-based search, which provides a rigorous (but computationally expensive)
approach to search for optimal controller parameters. We discretize the control space \( \mathcal{U} \) by using three points in each direction, which gives \( 3^3 = 81 \) possible controller parameters. This number decreased to 30 by imposing constraints \( w_{g_2} \leq w_{g_{25}} \leq w_{g_{ref}} \). For each controller setting, we perform 230 simulations to calculate the mean power output as well as the 50-year and 1-year characteristic loads using the peak-over-threshold approach reported in the IEC-61400 standard. The results obtained with the standard torque controller are shown in Table I. The simulations are parallelized by using 23 computing cores. Each point shown in the table corresponds to 230 simulations, and takes about 12 minutes of wall clock time using 23 computing cores. We use a multi-core computing server with Intel(R) Xeon(R) CPU E5-2698 v3 processors running at 2.30GHz. The entire set of results reported in the table require 5.8 hours of wall-clock time. We have calculated the sensitivity of the estimation of the mean power as well as of the 50-year threshold on the number of scenarios. We have found that around 15-20 simulations per bin are needed to estimate these quantities reliably.

From Table I we see that, given the load constraint \( \tilde{y}_L(\alpha_{50yr}) \leq \bar{y}_L = 200 \text{ MNM} \), the optimal controller parameters are \( P_{\text{rated}} = 6 \text{ MW}, \ w_{g_{ref}} = 135.20 \text{ rad/sec}, \ w_{g_2} = 117.82 \text{ rad/sec}, \ w_{g_{25}} = 133.85 \text{ rad/sec} \). The corresponding optimal expected power is 3.08 MW and the characteristic load is \( \tilde{y}_L(\alpha_{50yr}) = 196 \text{ MNM} \). At an average electricity price of 30 USD/MWh, the total revenue collected by the turbine is 809,000 USD/yr (a total extracted power of 27 GW h/yr). In the first row of Table I we present the performance of the turbine under base control parameters. Under these parameters, the expected power is 2.64 MW and the load is 166 MNM. These base parameters yield an average annual revenue of 693,000 USD/yr (a total extracted power of 23 GW h/yr). The relative improvement is approximately 17.4% and we note that the base control parameters are conservative relative to the load threshold. These results highlight that the control parameters have a significant effect on the economic and structural performance of the turbine. From Table I we can also observe that increasing \( P_{\text{rated}} \) and \( w_{g_{ref}} \) increase both expected power and the characteristic load, while increasing \( w_{g_{25}} \) decreases both expected power and the characteristic load. Increasing \( w_{g_2} \) decreases both expected power and characteristic load except when \( P_{\text{rated}} = 5 \text{ MW} \) and \( w_{g_{ref}} = 147.49 \text{ rad/sec} \), in which case the torque controller in region 2 becomes steeper than in region 2.5. This indicates non-monotonic behavior and highlights the need to develop advanced search procedures for control settings.

### B. Optimization-Based Search

The all-at-once optimization-based approach can bypass the need to simulate the wind turbine model repetitively and can handle a wider range of degrees of freedom. To implement this approach, we generate wind profiles according to the IEC-61400 standard and discretize the dynamic model using a third-order Radau collocation scheme [16]. Experiments show that to achieve the same level of accuracy, using Radau collocation scheme needs significantly less time steps than using explicit Euler method. To accurately capture extreme loads in the block maxima method, we have found that it is necessary to discretize the model using a resolution of 0.5 seconds over 10 minutes, giving rise to 1,200 time steps. The stochastic optimization problem is a large-scale nonlinear program (NLP). For 230 scenarios, the total number of variables in the NLP is 7.5 million. The stochastic optimization problem is highly nonlinear and the minimum and maximum functions in the control laws introduce numerical difficulties. In particular, selecting a smoothing parameter \( \epsilon \) is nontrivial (a large value would cause solver difficulty while small value results in low fidelity). In this work, we set \( \epsilon = 1 \). Table II shows that the optimization-based approach can reduce the computational time of the simulation-based search by a factor of two to four. We validated the solution of the optimization-based search (which includes smoothed control laws and time discretization) by fixing the controller parameters obtained and running high-fidelity dynamic simulations for the 230 scenarios. Such simulations are then used to compute expected power and the characteristic load with the peak-over-load threshold method.

### Table I: Expected power output and 1-yr and 50-yr characteristic loads for simulation-based search (using standard torque control law).

<table>
<thead>
<tr>
<th>( P_{\text{rated}} ) (MW)</th>
<th>( w_{g_{ref}} ) (rad/sec)</th>
<th>( w_{g_2} ) (rad/sec)</th>
<th>( w_{g_{25}} ) (rad/sec)</th>
<th>( \hat{y}_P ) (MW)</th>
<th>( \hat{y}<em>L(\alpha</em>{50yr}) ) (MNM)</th>
<th>Solution Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>122.91</td>
<td>117.82</td>
<td>121.68</td>
<td>2.64</td>
<td>166</td>
<td>144</td>
</tr>
<tr>
<td>5.0</td>
<td>135.20</td>
<td>117.82</td>
<td>133.85</td>
<td>2.76</td>
<td>187</td>
<td>160</td>
</tr>
<tr>
<td>5.0</td>
<td>147.49</td>
<td>117.82</td>
<td>146.02</td>
<td>2.79</td>
<td>205</td>
<td>175</td>
</tr>
<tr>
<td>6.0</td>
<td>122.91</td>
<td>117.82</td>
<td>121.68</td>
<td>2.83</td>
<td>208</td>
<td>176</td>
</tr>
<tr>
<td>6.0</td>
<td>135.20</td>
<td>117.82</td>
<td>133.85</td>
<td>3.08</td>
<td>225</td>
<td>189</td>
</tr>
<tr>
<td>6.0</td>
<td>147.49</td>
<td>117.82</td>
<td>146.02</td>
<td>3.18</td>
<td>226</td>
<td>190</td>
</tr>
</tbody>
</table>

### Table II: Results for simulation-based and optimization-based search using standard torque control law.

<table>
<thead>
<tr>
<th>Approach</th>
<th>( P ) (MW)</th>
<th>( P_{\text{rated}} ) (MW)</th>
<th>( w_{g_{ref}} ) (rad/sec)</th>
<th>( w_{g_2} ) (rad/sec)</th>
<th>( w_{g_{25}} ) (rad/sec)</th>
<th>( \hat{y}_P ) (MW)</th>
<th>( \hat{y}<em>L(\alpha</em>{50yr}) ) (MNM)</th>
<th>Solution Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>6</td>
<td>135.20</td>
<td>117.82</td>
<td>133.85</td>
<td>121.68</td>
<td>3.08</td>
<td>196</td>
<td>5.8</td>
</tr>
<tr>
<td>Optimization</td>
<td>3.05</td>
<td>6</td>
<td>127.26</td>
<td>117.82</td>
<td>121.68</td>
<td>3.11</td>
<td>194</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Our results indicate that, while both the exhaustive search and optimization-based search push $P_{\text{rated}}$ to its largest value of $P_{\text{rated}} = 6 \text{ MW}$ and push $w_{g_2}$ its smallest value of $w_{g_2} = 117.82 \text{ rad/sec}$, the settings for $w_{g_{ref}}$ and $w_{g_{ref}} = 127.83 \text{ rad/sec}$ are quite different. The expected power obtained with optimization is 1% larger than that of the simulation-based search while the characteristic load of settings from optimization is about 1% lower than that of the exhaustive search. Figure 4 shows an instance for power and load profiles obtained with optimization, high-fidelity simulation with optimal controller settings, and high-fidelity simulation with nominal settings. Compared with the simulation approach with nominal settings, the optimization approach clearly increases power output and the load. We also observe that the profiles obtained with optimization and high-fidelity validation match well.

Based on the optimal parameters obtained with optimization, we have derived a heuristic search. This consists of keeping both $w_{g_2}$ and $w_{g_{ref}}$ at the nominal value, and increasing $P_{\text{rated}}$ and $w_{g_{ref}}$. With only two degrees of freedom, we can discretize the space $\mathcal{U}$ using a finer mesh. Both $w_{g_{ref}}$ and $P_{\text{rated}}$ are discretized with 6 points, resulting in $6^2 = 36$ combinations. The results are shown in Table III from where we can conclude that the optimal setting $P_{\text{rated}}$ is 6 MW and the optimal setting $w_{g_{ref}}$ is 127.83 rad/sec. Compared with the nominal, the optimal settings increase the power output from 2.64 MW to 3.12 MW (improvement of 18%) and $\bar{y}_L(\alpha_{50\text{yr}})$ from 166 MNm to 196 MNm (increase of 18%). This illustrates that the optimization-based approach can be used to identify more effective heuristic search strategies. From these results we also observe that the characteristic load is more sensitive to the changes in $w_{g_{ref}}$ than the changes in $P_{\text{rated}}$. For example, increasing $P_{\text{rated}}$ from 5 MW to 6 MW increases the expected power from 2.64 MW to 2.94 MW at the cost of increasing $\bar{y}_L(\alpha_{50\text{yr}})$ from 166 MNm to 175 MNm. However, increasing $w_{g_{ref}}$ from 122.91 rad/sec to 147.49 rad/sec only increases the expected power 2.64 MW to 2.85 MW while increases $\bar{y}_L(\alpha_{50\text{yr}})$ from 166 MNm to 225 MNm. The 1-year characteristic load $\bar{y}_L(\alpha_{1\text{yr}})$ is notably smaller than $\bar{y}_L(\alpha_{50\text{yr}})$. If we relax the load constraint to $\bar{y}_L(\alpha_{1\text{yr}}) \leq \bar{y}_L$, then the optimal $w_{g_{ref}}$ increases to 137.66 rad/sec and the optimal power increases from 3.12 MW to 3.19 MW (an improvement of 2%).

Table IV presents results obtained with the modified torque control law. Here, we use the proposed heuristic search to identify optimal parameters. By comparing with the results obtained with the standard control law (shown in Table III) we observe that, for almost all combinations of $P_{\text{rated}}$ and $w_{g_{ref}}$, the change of control law has limited effect on the load but can increase the mean power output. In particular, for a load constraint $\bar{y}_L(\alpha_{50\text{yr}}) \leq \bar{y}_L = 200\text{ MNm}$, the optimal power increases from 3.12 MW to 3.24 MW. Table V confirms that the controller parameters obtained with the exhaustive and optimization search for the modified control law coincide.

The different results obtained with standard and modified torque controllers highlight that the control law has a significant effect on economic and structural performance. To understand how strong this effect can actually be, we solved an optimization instance without control laws. That is, the time profile of the angle $\theta(t)$ and torque $T_{\text{gen}}(t)$ are used directly as decision variables. This can be interpreted as an MPC control strategy. We note that this induces a large number of degrees of freedom (in this case 2,400). Consequently, a simulation-based search is no longer applicable. Table VI shows that the expected power obtained with MPC is 3.9 MW, which represents an improvement of 25.9% relative to the optimal settings found with the use of control laws (3.07 MW). The MPC strategy improves the annual power by 7 GWh/yr (at an electricity price of 30 USD/MWh this represents and improvement of 218,000 USD/yr). We also note that the solution times of the MPC strategy are reduced to 40 minutes, which highlight the complexity introduced by the non-smooth control laws. The solution time of 40 minutes is too long to enable real-time MPC implementations. The deployment of MPC would require simplification of the formulation (e.g. less time steps and less scenarios). Although the deployment of MPC is more challenging, these results provide an idea of the potential performance. These results also highlight that the proposed framework can be used as a research tool to investigate the benefits of different control architectures. Moreover, the proposed stochastic optimization framework can also be used within an MPC setting to perform long-term statistical extrapolation.
We proposed stochastic optimization formulations to mitigate long-term extreme loads in controller design procedures. The proposed formulation incorporates a rigorous procedure to estimate the probability of long-term extreme load exceedance (as described by the IEC-61400 standard). We provide evidence that the approach can significantly improve performance of nominal controller parameters. We also demonstrate that the formulation can be solved efficiently by using parallel optimization solvers.

This paper presents a general methodology and demonstrates it using a simplified wind turbine model and control laws. We acknowledge that in order to enable industrial implementations of the proposed approach, more sophisticated studies need to be performed. Such studies should incorporate detailed models that capture fatigue and extreme loads on all components. Moreover, it is important to consider aspects of the generator and power electronics, more complex control configurations that handle shut down procedures and supervisory control, and more complex wind conditions (turbulent flow fields and wind gusts).

V. CONCLUSIONS AND FUTURE WORK
$c_T = 17782$ : Tower structural damping [kg/s]

$k_T = 1810000$ : Bending stiffness [kg/s$^2$]

$P_{\text{rated0}} = 5$ : Nominal rated power [MW]

$w_{\text{rated0}} = 1.267$ : Nominal rated rotor speed [rad/sec]

$w_{\gamma_{\text{ref}}} = 122.91$ : Nominal reference generator speed [rad/sec]

$w_{\gamma_{\text{rated}}} = 121.68$ : Nominal boundary speed [rad/sec]

$V_{\text{rated}} = 11.2$ : Rated wind speed [m/sec]

$\lambda_{\text{opt}} = 8.22$ : Optimal tip speed ratio [-]

$cp_{\text{max}} = 0.51$ : Peak power coefficient [-]

$\theta_{\text{min}} = 0$ : Minimum blade pitch angle [°]

$\theta_{\text{max}} = 30$ : Maximum blade pitch angle [°]

### References


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