Stochastic Optimization Formulations for Wind Turbine Power Maximization and Extreme Load Mitigation

Yankai Cao, Fernando D'Amato, and Victor M. Zavala

***Abstract*—We propose stochastic optimization formulations to identify optimal parameters for pitch and torque controllers in wind turbines. The approach seeks to extract maximum power and to satisfy extreme mechanical load requirements by design. The proposed formulation incorporates a wind turbine dynamic model, control law representations for pitch and torque controllers, and a probabilistic constraint that captures the long-term probability of exceeding an extreme load threshold (as described by the IEC-61400). We use the fact that the extreme load follows a generalized extreme value distribution to obtain an explicit algebraic characterization of the probabilistic constraint. The proposed approach can find controller parameters that increase power output relative to a nominal control design without changing the structure of existing controllers and while maintaining the original extreme load characteristics. We also demonstrate that the optimization formulation, which is cast as a large-scale nonlinear programming problem with up to 7.5 million variables and constraints, can be solved in less than 1.5 hours using a state-of-the-art parallel interior-point solver.**

***Index Terms*—Stochastic, optimization, fatigue, wind turbines.**

I. INTRODUCTION

INDUSTRIAL wind turbines are designed to operate through a lifetime of more than 15 years subject to uncertain weather. Strong wind conditions that are expected to happen during this lifetime could compromise the turbine's integrity, if not properly handled through the control system. To prevent structural damage and high life consumption rates, the International Electrotechnical Commission (IEC) standards require wind turbine designers to ensure that the turbine does not exceed critical load conditions when subject to a multiplicity of hypothetical operating scenarios. Many of these critical load conditions are derived using statistical load extrapolation methods [1], [2], [3] that seek to use limited wind data to assess the long-term probability of the turbine of exceeding a certain load threshold. Extrapolation techniques are based on a powerful result from extreme value theory that states that the maximum of a sequence of independent random variables follows a generalized extreme value distribution. The existence of such extreme value distribution enables the estimation of probabilities of rare long-term events.

Controlling the operation of an industrial wind turbine requires a careful tradeoff between energy capture and compliance with structural loads. Control strategies for industrial wind turbines are typically based on control laws that regulate power and rotor speed by operating generator torque and collective blade pitch angle. In addition, multiple control laws are added to mitigate loads [4], [5]. As a result, designing a viable controller involves iterative tuning multiple control parameters until it satisfies the requirements of the IEC standard. This process is tedious and computationally expensive.

Recent research activity [6], [7], [8], [9], [10] has been focusing on Model Predictive Control (MPC) technology to handle regulation controls for power and speed as well as time domain constraints typical from the IEC standard. MPC is a powerful optimization-based control technology that can aid standard controllers, as it can directly accommodate detailed turbine models and constraints of different forms and with this anticipate wind events and capture multivariable interactions. In particular, MPC strategies can perform simultaneous blade pitch and generator torque control while maximizing power and mitigating extreme loads. For instance, in the work of [6], it is shown that under an extreme gust event, an MPC strategy can reduce the tower base moment by up to 15% compared to standard controllers. Similarly, the work of [7] uses a nonlinear MPC formulation to demonstrate that some loads can be reduced by up to 50% under extreme gusts without negative impact on overall power production. The nonlinear MPC formulation of [10] also shows that it can reduce the tower bending moment by up to 40%. Simplified MPC strategies have also been reported in the literature that seek to overcome computational complexity by using linearized model representations [8], [9]. A limitation of MPC strategies reported in the literature is that they analyze robustness to diverse wind events on a case by case basis and do not capture long-term extreme load constraints as required by IEC standards. Unfortunately, the computational procedures needed to fit operational loads to extreme value distributions and to predict long-term extreme loads are complex and not trivial to implement in MPC formulations. Because of this, the effect of control strategies on wind turbine loads is often performed *a posteriori* and not *a priori* by design.

The main contribution of this work is the proposal of a stochastic optimization formulation to obtain pitch and torque controller parameters that maximize power while enforcing extreme load constraints as required by IEC standards. The formulation incorporates a simplified nonlinear wind turbine model and extracts load maxima from multiple wind field scenarios to estimate a long-term probability of load threshold violation. In addition, the formulation is used to evaluate a best hypothetical MPC supervisory strategy, that can be used as a reference metric for the maximum achievable performance. The methodology is benchmarked against simplified control strategies proposed in the literature [10], [11] that define parameters for the pitch and torque control laws operating mostly at rated conditions. We also show that the optimization

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formulation, which is cast as a large-scale nonlinear program with up to 7.5 million variables, can be solved in less than 1.3 hours by using a state-of-the-art parallel solver. We also provide evidence that the approach can find controller parameters that increase power output relative to a nominal control design without changing the structure of existing controllers and while maintaining the original extreme load characteristics.

II. Optimization Formulations

In this section, we present a deterministic and a stochastic formulation for the wind turbine power maximization problem. The goal in the deterministic formulation is to calculate optimal controller parameters that maximize the wind turbine power. We also impose constraints on the maximum load experienced by the turbine under a given known wind speed profile. In the stochastic optimization formulation, the goal is to calculate optimal controller parameters that maximize expected power over an uncertain wind power profile (where uncertainty is captured in the form of scenarios). The formulation also imposes probabilistic constraints for the exceedance of given load thresholds in the long-term loads. The probabilistic constraint uses the methodology required by the IEC61400-1 standard. The formulations also capture physical dynamics of the turbine and incorporate controller laws. Appendix A describes all variables, parameters, and units of the model.

A. *Wind Turbine Model*

The wind turbine dynamics are described by the following system of differential and algebraic equations (DAEs) [10]:

$$\dot{w_r}(t) = \frac{1}{J}(M_z(t) - N_g T_{gen}(t)) \tag{II.1a}$$
$$\dot{x}(t) = v_x(t) \tag{II.1b}$$
$$\dot{v_x}(t) = \frac{1}{m_{Te}}(-c_{Te}v_x(t) - k_{Te}x(t) + F_z(t)) \tag{II.1c}$$
$$V_{rel}(t) = V(t) - v_x(t) \tag{II.1d}$$
$$\lambda(t) = w_r(t)R_r/V_{rel}(t) \tag{II.1e}$$
$$C_t(t) = \text{Thrust}(\theta(t), \lambda(t)) \tag{II.1f}$$
$$C_m(t) = \text{Torque}(\theta(t), \lambda(t)) \tag{II.1g}$$
$$F_z(t) = \frac{1}{2}\rho V_{rel}(t)^2 A C_t(t) \tag{II.1h}$$
$$M_z(t) = \frac{1}{2}\rho V_{rel}(t)^2 A R_r C_m(t)/\lambda(t) \tag{II.1i}$$
$$y_P(t) = T_{gen}(t)w_g(t)(1 - P_1) \tag{II.1j}$$
$$y_L(t) = H(k_{Te}x(t) + c_{Te}v_x(t)) \tag{II.1k}$$

Equations (II.1f) and (II.1g) are highly nonlinear relationships and re given in [10].

The wind turbine is equipped with *single-loop* controllers for torque $T_{gen}(t)$ and pitch angle $\theta(t)$. The *generator torque controller* used has the form proposed in [11]. Given a set-point for rated power P_{rated} and the generator speed $w_g(t) = w_r(t)N_g$, the controller adjusts the torque according to:

$$T_{gen}(t) = \begin{cases} \kappa w_g(t)^2, & \text{if } w_g(t) \leq w_{g_2} \\ \kappa w_{g_2}^2 + \frac{w_g(t) - w_{g_2}}{w_{g25} - w_{g_2}}\left(P_{rated}/w_{g_{25}} - \kappa w_{g_2}^2\right), & \text{if } w_{g_2} < w_g(t) \leq w_{g_{25}} \\ P_{rated}/w_g(t), & \text{if } w_g(t) > w_{g_{25}} \end{cases} \tag{II.2}$$

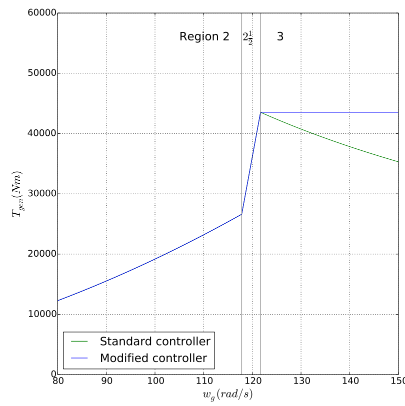

Fig. 1: **Standard and modified torque control laws.**

where $\kappa > 0$ is a parameter. As shown in Fig. 1, the control law is piecewise linear (non-smooth) and has three regions of interest. The parameter w_{g_2} denotes the threshold between region 2 and region 2.5 and $w_{g_{25}}$ denotes the threshold between region 2.5 and region 3. This strategy defines the torque setpoints in region 3 to provide rated power. For the *blade pitch controller*, we use a proportional-integral control law [11] that adjusts the pitch angle given a set-point for the generator speed ($w_{g_{ref}}$):

$$\Delta w_g(t) = w_g(t) - w_{g_{ref}} \tag{II.3a}$$
$$w_{g_{int}}(t) = \int_0^t \Delta w_g(\tau)d\tau \tag{II.3b}$$
$$w_{g_{int}}(t) = \max(w_{g_{int}}(t-1) + \Delta w_g(t)h, 0) \tag{II.3c}$$
$$\theta(t) = \max(K_P \Delta w_g(t) + K_I w_{g_{int}}(t), 0) \tag{II.3d}$$

and we use the parameters $K_P = 0.216$, $K_I = 0.0924$. By using the control laws described before, the optimization problem has four degrees of freedom, given by the *pitch and torque controller parameters* (set-points) $u = (P_{rated}, w_{g_{ref}}, w_{g_2}, w_{g_{25}})$. The parameters P_{rated}, w_{g_2}, and $w_{g_{25}}$ determine the performance of the local controller for torque, and $w_{g_{ref}}$ determines the performance of the pitch controller. In this work we use a modified torque control law, which allows power to go above rated values during short time intervals:

$$T_{gen}(t) = \begin{cases} \kappa w_g(t)^2, & \text{if } w_g(t) \leq w_{g_2} \\ \kappa w_{g_b}^2 + \frac{w_g(t) - w_{g_b}}{w_{g25} - w_{g_b}}\left(P_{rated}/w_{g_{25}} - \kappa w_{g_b}^2\right), & \text{if} w_{g_2} < w_g(t) \leq w_{g_{25}} \\ P_{rated}/w_{g_{25}}, & \text{if } w_g(t) > w_{g_{25}}. \end{cases} \tag{II.4}$$

Here, instead of maintaining constant power in region 3, we adopt the strategy proposed in [10] to maintain the rated torque value whenever the rotor speed exceeds its rated value. We emphasize that the only difference between the modified and the standard control law is in region 3, as shown in Figure 1. In particular, in region 3 of the modified formulation, the generator torque remains constant as $w_g(t)$ increases, and thus the power output increases as $w_g(t)$ increases. In Section IV we show that this modification increases performance.

B. Deterministic Optimization Formulation

We represent the deterministic wind turbine optimization problem in the following abstract form:

$$\max_{u\in\mathcal{U}} \frac{1}{T}\int_{\mathcal{T}} y_P(t)dt \tag{II.5a}$$

$$\text{s.t. } (y_P(t), y_L(t)) = \phi(u, V(t)),\ t\in\mathcal{T} \tag{II.5b}$$

$$y_L(t) \leq \bar{y}_L,\ t\in\mathcal{T} \tag{II.5c}$$

where $t \in \mathcal{T} := [0, T]$, $y_P(t)$ is the wind turbine power, $y_L(t)$ is the mechanical load with associated threshold $\bar{y}_L$, and $V(t)$ is the wind profile. The controls u (the controller parameters) are restricted to the closed set $\mathcal{U}$, which is a four-dimensional hypercube defined by the following constraints:

$$P_{rated0} \leq P_{rated} \leq 1.2P_{rated0} \tag{II.6a}$$

$$w_{g_{ref0}} \leq w_{g_{ref}} \leq 1.2w_{g_{ref0}} \tag{II.6b}$$

$$w_{g_{250}} \leq w_{g_{25}} \leq 1.2w_{g_{250}} \tag{II.6c}$$

$$w_{g_{20}} \leq w_{g_2} \leq 1.2w_{g_{20}}. \tag{II.6d}$$

An important practical problem is that power maximization often conflicts with the mechanical load experienced by the turbine (i.e., the higher the power extracted the higher the load). Consequently, it is critical to carefully trade-off these metrics so as to prevent putting the turbine at mechanical risk.

The optimization formulation is a *controller design* formulation that seeks to identify optimal parameters for the low-level torque and pitch controllers and allow the low-level controllers to act the pitch angle and torque to react to high-frequency wind variations. Additionally, we consider the problem of allowing MPC to directly compute the control actions on pitch and torque actuation, without the low level control loops. This approach although not suitable for practical implementation, can provide valuable information about achievable performance. We discuss this more advanced setting in Section IV.

C. Standard Stochastic Optimization Formulation

To account for wind variability (see Figure 3), we use a stochastic formulation. We model the wind profile as a random variable $V(t)$ and use $V(t,\omega)$ to denote a realization $\omega \in \Omega$ obtained from the probability density $p(V(t))$. Here, Ω represents a scenario (or realization) set. We note that, because the wind profile propagates through the wind turbine model, the turbine extracted power and load outputs are also random variables. To maintain notational consistency, the random power output is denoted simply as $y_P(t)$ with realizations $y_P(t,\omega)$ and the random load is $y_L(t)$ with realizations $y_L(t,\omega)$. The *posterior probability density* of the power output and for the load are denoted as $p_P(y_P(t)\,|\,V(t),u)$ and $p_L(y_L(t)\,|\,V(t),u)$. Realizations from these posterior densities (denoted as $y_P(t,\omega)$ and $y_L(t,\omega)$) are obtained by propagating the wind speed realizations $V(t,\omega)$ and u through the model. The posterior densities *do not have a closed form* due to the nonlinearity of the wind turbine model.

A standard stochastic formulation maximizes the expected power $\mathbb{E}\left[\frac{1}{T}\int_{\mathcal{T}} y_P(t,\omega)dt\right]$ while imposing a probabilistic constraint of the form:

$$\mathbb{P}\left\{y_L(t,\omega) \leq \bar{y}_L,\ t\in\mathcal{T}\right\} \geq \alpha. \tag{II.7}$$

where α is a probability level close to one. The probabilistic constraint (II.7) can also be expressed as $\mathbb{P}\left\{\max_{t\in\mathcal{T}} y_L(t,\omega) \leq \bar{y}_L\right\} \geq \alpha$. This constraint enforces that the probability that the maximum load over the time horizon $\mathcal{T}$ exceeds the threshold is no more than $1-\alpha$. Moreover, the cumulative density of the extreme load $\max_{t\in\mathcal{T}} y_L(t,\omega)$ is denoted as $F_{max}(\hat{y}_L(\alpha)) = \mathbb{P}\left\{\max_{t\in\mathcal{T}} y_L(t,\omega) \leq \hat{y}_L(\alpha)\right\}$, where $\hat{y}_L(\alpha)$ is the α-critical value (fractile/quartile) of the distribution. We refer to this critical load value as the *characteristic load*. The probabilistic constraint can also written as $F_{max}(\hat{y}_L(\alpha)) \geq \alpha$ or one can use the constraint on the quartile $\hat{y}_L(\alpha) \leq \bar{y}_L$. In particular, we note that $\hat{y}_L(\alpha) \leq \bar{y}_L$ implies $\mathbb{P}\left\{\max_{t\in\mathcal{T}} y_L(t,\omega) \leq \hat{y}_L(\alpha)\right\} \geq \alpha$.

The probabilistic load constraint (II.7) is enforced over the time domain $\mathcal{T}$. Clearly, if rare and extreme load scenarios over horizons of years need to be captured, this approach would be computationally impractical. Moreover, the conditional density $F_{max}(\cdot)$ does not have a closed form due to the nonlinearity of the wind turbine model. We now describe a procedure to approximate this long-term probabilistic constraint by using statistical extrapolation methods.

D. Stochastic Formulation Based on Statistical Extrapolation

The IEC-61400 standard proposes a procedure to enforce long-term (1-year and 50-year) probabilistic constraints on extreme loads. This is done by using existing short-term load data to perform long-term extrapolation [1] and gives a special type of *probabilistic load constraint* that can be implemented computationally. This method has been shown to provide accurate estimates of long-term extreme loads [2], [3].

In this section we provide a summary of load extrapolation procedures. The key idea behind load extrapolation is to define a sufficiently long horizon (that we also define as $\mathcal{T}$ with some abuse of notation) under which the extreme load $\max_{t\in\mathcal{T}} y_L(t,\omega)$ achieves statistical steady-state and with this ensure that it becomes statistically independent of the extreme load at a subsequent time horizon $\mathcal{T}'$. With this, we can split a long horizon of interest into a set of short time horizons and define the extreme load over the long horizon as the maximum of the extreme loads over the short horizons (because the maximum loads over the short horizons are statistically independent). The length of the short horizon is typically chosen to be 10 minutes [1]. Load extrapolation methods are based on the crucial observation that the maximum of a set of independent random variables distributes according to the generalized extreme value distribution (here we focus on the Gumbel distribution, which is a special case).

The *peak-over-threshold* procedure to estimate the characteristic extreme loads is summarized as follows:

- Each realization of the wind $V(t,\omega),\ t \in \mathcal{T}$ has an associated time-average wind speed $\bar{V}$. We discretize the time-average wind-speed range into $j \in \mathcal{B} := \{1, ..., B\}$ bins (the bins typically span the range $[3, 25]$ m/s and we use B =23 bins). We categorize every realization of the wind into a single bin j with corresponding mean wind speed $\bar{V}_j$ and create scenario sets Ω_j with $\Omega = \cup_{j\in\mathcal{B}}\Omega_j$.
- For a given wind speed bin $\bar{V}_j$, we gather wind turbine power realizations under fixed controller parameters u

and N_j wind profiles from the scenario set Ω_j. These realizations have associated loads $y_L(t,\omega),\ \omega \in \Omega_j$ from which we extract a total of n_j local load maxima that are contained in a vector $L_j \in \Re^{n_j}$. The maxima are defined as all local load maxima above a given threshold $\hat{L}_j$.

- We fit the local maxima to a Gumbel distribution. The parameters of the distribution are $c_j = \sqrt{6}\,\sigma_j\ \pi$ and $L_j^0 = \overline{L}_j - c_j\gamma$ for $j \in \mathcal{B}$. Here, σ_j is the standard deviation of L_j, $\overline{L}_j$ is the mean of L_j, and γ is the Euler constant.
- We compute the long-term exceedance probability for the load $P_{ex}(y_L)$ by integrating over all operating wind speeds. This is done by using the equations $F_j(y_L) = \exp - \left(\exp - \left((y_L - L_j^0)/c_j\right)\right),\ j \in \mathcal{B}$ and $P_{ex}(y_L) = \sum_{j\in\mathcal{B}} p_j(1 - F_j(y_L)^{n_j})$. Here, $F_j(\cdot)$ is cumulative Gumbel density (probability that the load is below the critical value y_L) conditional to the mean speed $\bar{V}_j$ and p_j is the probability that the mean wind speed is within the bin $j \in \mathcal{B}$. The probabilities can be computed by assuming that the mean wind speed follows a Rayleigh distribution.
- We solve the equation $P_{ex}(y_L) = \alpha_{50yr}$ with $\alpha_{50yr} = \frac{10}{50\times365\times24\times60} = 3.8\times10^{-7}$ for y_L to obtain the 50-year *characteristic load* $\hat{y}_L(\alpha_{50yr})$ or solve $P_{ex}(y_L) = \alpha_{1yr}$ with $\alpha_{1yr} = \frac{10}{1\times365\times24\times60} = 1.9\times10^{-5}$ for y_L to obtain the 1-year load $\hat{y}_L(\alpha_{1yr})$.

The characteristic load $\hat{y}_L(\alpha)$ is the critical load (a fractile) that exceeds the threshold $\bar{y}_L$ with probability α. The 50-year characteristic load is to be interpreted as the load that, on average, is *exceeded only once every 50 years*. Similarly, the 1-year load is to be interpreted as the load that, on average, is exceeded only once every one year.

In a simulation setting, we obtain wind turbine realizations of the loads $y_L(t,\omega)$ by performing model simulations. The IEC-61400 load extrapolation procedure can thus be represented by using an input-output function of the form $\hat{y}_L(\alpha) = \varphi(y_L(\cdot,\cdot))$ where $\hat{y}_L(\alpha)$ is the characteristic load at probability level α (e.g., 1-year, 20-year, 50-year). The load extrapolation procedure allows us to impose a constraint on the characteristic load of the form $\hat{y}_L(\alpha) \le \bar{y}_L$. This constraint on the fractile $\hat{y}_L(\alpha)$ implies that the probability that the load exceeding the threshold $\bar{y}_L$ is less than $1-\alpha$ (i.e., the probability that the load is below $\bar{y}_L$ is greater than α).

III. Solution of Stochastic Formulation

In this section we propose simulation-based and optimization-based (all-at-once) procedures to solve the stochastic optimization formulation.

A. Exhaustive Simulation-Based Search

The number of degrees of freedom u in the stochastic formulation is small (in our case these correspond to the controller parameters). Consequently, it is possible to perform an exhaustive search of the space $u \in \mathcal{U}$. This approach has the advantage that it allows us to explore the whole decision space and provides information on how power and maximum loads change with the controller parameters. Moreover, this approach is parallelizable and does not require derivative information. Consequently, with this approach we can handle non-smooth functions associated to the pitch and torque control laws and we can directly implement the IEC-61400 procedure. On the other hand, even when parallel simulations are performed, an extremely large number of simulations will be needed to span the entire decision space and the method will not scale. Scalability issues will also become evident when more degrees of freedom need to be considered (as in MPC formulations). The simulation-based search procedure follows the basic steps:

1) Given α, for each $u \in \mathcal{U}$, DO:
2) For each scenario wind scenario $V(t,\omega)$, simulate the dynamic model equations and compute expected power $\mathbb{E}\left[\frac{1}{T}\int_{\mathcal{T}} y_P(t,\omega)dt\right]$ and loads $y_L(t,\omega)$.
3) Use IEC-61400 load extrapolation procedure to match observed extreme load moments to moments of distribution.
4) Use moments to compute characteristic load $\hat{y}_L(\alpha)$.

After exploration of the entire control space $u \in \mathcal{U}$, determine the combination that maximizes expected power and satisfies the characteristic load constraint $\hat{y}_L(\alpha) \le \bar{y}_L$.

B. Optimization-Based Search

In an all-at-once optimization approach, we discretize the wind turbine model to obtain a purely algebraic representation of the wind turbine model and with this we bypass the need to simulate the model repetitively. Another benefit of this approach is that we can obtain derivative information and handle a wider range of degrees of freedom and constraints (e.g., to consider different parameters in which an MPC controller overrides low-level controllers). The presence of non-smooth pitch and torque controller laws, however, requires of suitable smooth reformulations. Moreover, the load IEC-61400 extrapolation procedure requires an automated procedure to extract multiple local maxima from load profiles to fit the long-term extreme value distribution. This is inherently a *dual control problem* that seeks to maximize power while maximizing information extraction to perform long-term load inference. We now describe strategies to address these issues.

To ensure differentiability, the $\min$ and $\max$ functions used in the control laws are approximated using sigmoidal smooth functions. In particular, for a function of the form $\max\{x,y\}$ is approximated using the function $x + \frac{1}{\epsilon}\log(1+e^{-\epsilon(x-y)})$ while a function of the form $\min\{y,x\}$ can be approximated using $x - \frac{1}{\epsilon}\log(1+e^{-\epsilon(y-x)})$. Here, $\epsilon > 0$ is a smoothing parameter that controls the quality of the approximation.

To implement the IEC-61400 procedure in an all-at-once optimization setting, we use the so-called *block maximum* approach instead of the *peak-over-threshold* approach described before. Here, we partition the horizon in $\mathcal{S} := \{1..S\}$ stages of length T/S and extract the maximum load in each stage. The sets $\mathcal{T}_s$ is the time domain of stage s and satisfy $\cup_{s\in\mathcal{S}}\mathcal{T}_s = \mathcal{T}$. We extract a total of S load maxima per wind realization and we denote the maximum load in stage $s \in \mathcal{S}$ and realization ω as $\eta_s(\omega)$. We recall that the maximum loads are categorized by wind speed level by splitting the realization set into subsets $\Omega_j,\ j \in \mathcal{B}$. Figure 2a illustrates how the *peak-over-threshold* and *block maximum* approaches select peak load values. Figure 2b compares the characteristic load obtained using these approaches for different number of stages. The

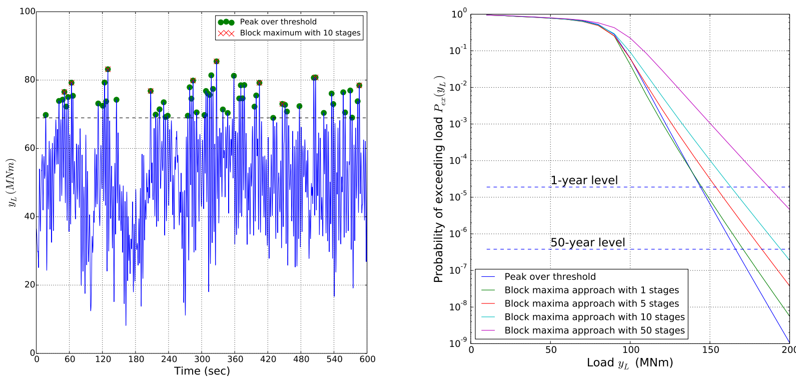

Fig. 2: **Extreme loads (top) and extreme value distributions (bottom) for peak over threshold and block maxima approaches.**

characteristic load obtained with the block maximum method is close to that of the peak-over-threshold method.

$$\min_{u\in\mathcal{U}} \sum_{s\in\mathcal{S}} \mathbb{E}\left[\eta_s(\omega)\right] \quad \text{(III.8a)}$$

$$\text{s.t. } (y_P(t,\omega), y_L(t,\omega)) = \phi(u, V(t,\omega)),\ t\in\mathcal{T},\ \omega\in\Omega \quad \text{(III.8b)}$$

$$y_L(t,\omega) \le \eta_s(\omega),\ \omega\in\Omega, t\in\mathcal{T}_s,\ s\in\mathcal{S} \quad \text{(III.8c)}$$

$$0 \le \eta_s(\omega),\ \omega\in\Omega,\ s\in\mathcal{S} \quad \text{(III.8d)}$$

$$\overline{L}_j = \frac{1}{|\mathcal{S}||\Omega_j|}\sum_{s\in\mathcal{S}}\sum_{\omega\in\Omega_j}\eta_s(\omega),\ j\in\mathcal{B} \quad \text{(III.8e)}$$

$$\sigma_j^2 = \frac{1}{|\mathcal{S}||\Omega_j|}\sum_{s\in\mathcal{S}}\sum_{\omega\in\Omega_j}(\eta_s(\omega) - \overline{L}_j)^2,\ j\in\mathcal{B} \quad \text{(III.8f)}$$

$$c_j = \frac{\sqrt{6}\,\sigma_j}{\pi},\ j\in\mathcal{B} \quad \text{(III.8g)}$$

$$L_j^0 = \overline{L}_j - c_j\gamma,\ j\in\mathcal{B} \quad \text{(III.8h)}$$

$$F_j = \exp - \left(\exp - \left((\hat{y}_L - L_j^0)/c_j\right)\right),\ j\in\mathcal{B} \quad \text{(III.8i)}$$

$$P_{ex} = \sum_{j\in\mathcal{B}} p_j(1 - F_j^S) \quad \text{(III.8j)}$$

$$P_{ex} \le \alpha \quad \text{(III.8k)}$$

$$\hat{y}_L \le \bar{y}_L \quad \text{(III.8l)}$$

$$\mathbb{E}\left[\frac{1}{T}\int_{\mathcal{T}} y_P(t,\omega)dt\right] \ge \bar{P}. \quad \text{(III.8m)}$$

The formulation (III.8) implements the proposed block-maximum load extrapolation procedure. We treat this problem as a bi-objective problem that seeks to extract the maximum power while extracting maximum peak load information to perform long-term load extrapolation. We highlight that these objectives are conflicting because maximum power occurs at the maximum load. Because of this, the expected power is handled through the constraint (III.8m), where $\bar{P}$ is a minimum power level. The probability of exceedance $P_{ex}(y_L)$ is close to zero and this introduces numerical difficulties. To deal with this, we approximate $P_{ex}(y_L)$ using the Taylor expansion $P_{ex}(y_L) \approx \sum_{j\in\mathcal{B}} p_j(1 - F_j(y_L))$, which is much easier to handle computationally.

IV. Case Studies and Results

In this section we compare power and load profiles obtained with nominal and optimal controller parameters obtained with a simulation-based and an optimization-based search. The parallel simulation setting is implemented in `Julia` and the optimization setting is implemented in `PLASMO`, which is a `Julia`-based modeling framework that facilitates the construction and analysis of structured optimization models [12]. The structure of the problems is exploited using `PIPS-NLP`, which is an interior point solver that uses parallel Schur decomposition techniques to perform linear algebra [13]. Due to constraints on space, we only present a summary of the most relevant numerical results. A detailed version of this paper is available at http://zavalab.engr.wisc.edu.

A. Exhaustive Simulation-Based Search

We first show the results of simulation-based search, which provides a rigorous (but computationally expensive) approach to search for optimal controller parameters. We discretize the control space $\mathcal{U}$ by using three points in each direction, which gives $3^4 = 81$ possible controller parameters. This number decreased to 30 by imposing constraints $w_{g_2} \le w_{g_{25}} \le w_{g_{ref}}$. For each controller setting, we perform 230 simulations to calculate the mean power output as well as the 50-year and 1-year characteristic loads using the peak-over-threshold approach reported in the IEC-61400 standard. The results obtained with the standard torque controller are shown in Table I. The simulations are parallelized by using 23 computing cores. Each point shown in the table corresponds to 230 simulations, and takes about 12 minutes of wall clock time using 23 computing cores. We use a multi-core computing server with Intel(R) Xeon(R) CPU E5-2698 v3 processors running at 2.30GHz. The entire set of results reported on the table take 5.8 hours of wall clock time. We have calculated the sensitivity of the estimation of the mean power as well as of the 50-yr threshold on the number of scenarios. We have found that around 15-20 simulations per bin are needed to estimate these quantities reliably.

From Table I we see that, given the load constraint $\hat{y}_L(\alpha_{50yr}) \le \overline{y}_L = 200\,MNm$, the optimal controller parameters are $P_{rated} = 6\,MW, w_{g_{ref}} = 135.20\,rad/sec, w_{g_2} = 117.82\,rad/sec$, and $w_{g_{25}} = 133.85\,rad/sec$. The corresponding maximum expected power is $3.08\,MW$ and the characteristic load is $\hat{y}_L(\alpha_{50yr})$ =196 MNm. At an average electricity price of 30 USD/MWh, the total revenue collected by the turbine is 809,000 USD/yr (a total extracted power

TABLE I: **Expected power output and 1-yr and 50-yr characteristic loads for simulation-based search (using standard torque control law).**

P_{rated} (MW)	$w_{g_{ref}}$ (rad/sec)	w_{g_2} (rad/sec)	$w_{g_{25}}$ (rad/sec)	y_P (MW)	$\hat{y}_L(\alpha_{50yr})$ (MNm)	$\hat{y}_L(\alpha_{1yr})$ (MNm)
5.0	122.91	117.82	121.68	2.64	166	144
5.0	135.20	117.82	121.68	2.83	208	176
		117.82	133.85	2.76	187	160
		129.60	133.85	2.73	182	157
5.0	147.49	117.82	121.68	2.85	225	189
		117.82	146.02	2.79	205	175
		129.60	146.02	2.80	208	177
6.0	135.20	117.82	121.68	3.18	226	190
		117.82	133.85	3.08	196	170
		129.60	133.85	3.02	187	163
6.0	147.49	117.82	121.68	3.20	246	206
		117.82	146.02	3.13	216	185
		129.60	146.02	3.12	214	183

TABLE II: **Results for simulation-based and optimization-based search using standard torque control law.**

Approach	$\bar{P}$ (MW)	P_{rated} (MW)	$w_{g_{ref}}$ (rad/sec)	w_{g_2} (rad/sec)	$w_{g_{25}}$ (rad/sec)	y_P (MW)	$\hat{y}_L(\alpha_{50yr})$ (MNm)	Solution Time (hr)
Simulation		6	135.20	117.82	133.85	3.08	196	5.8
Optimization	3.05	6	127.26	117.82	121.68	3.11	194	1.5

of 27 GWh/yr. In the first row of Table I we present the performance of the turbine under base control parameters. Under these parameters, the expected power is $2.64MW$ and the load is 166 MNm. These base parameters yield an average annual revenue of 693,000 USD/yr (a total extracted power of 23 GWh/yr). The relative improvement is approximately 17.4% and we note that the base control parameters are conservative relative to the load threshold. These results highlight that the control parameters have a significant effect on the economic and structural performance of the turbine. From Table I we can also observe that increasing P_{rated} and $w_{g_{ref}}$ increase both expected power and the characteristic load, while increasing $w_{g_{25}}$ decreases both expected power and the characteristic load. Increasing w_{g_2} decreases both expected power and characteristic load except when $P_{rated} = 5\,MW$ and $w_{g_{ref}} = 147.49\,rad/sec$, in which case the torque controller in region 2 becomes steeper than in region 2.5. This indicates non-monotonic behavior and highlights the need to develop advanced search procedures for control settings.

B. Optimization-Based Search

The all-at-once optimization-based approach can bypass the need to simulate the wind turbine model repetitively and can handle a wider range of degrees of freedom. To implement this approach, we discretize the dynamic model using a Radau collocation scheme [14]. To accurately capture extreme loads in the block maxima method, we have found that it is necessary to discretize the model using a resolution of 0.5 seconds over 10 minutes, giving rise to 1,200 time steps. For an NLP with 230 scenarios, the total number of variables is 7.5 million. The models were implemented on `PLASMO` [12] and solved with `PIPS-NLP` [13]. The stochastic optimization problem is highly nonlinear and the minimum and maximum functions in the control laws introduce numerical difficulties. In particular, selecting an smoothing parameter ϵ is nontrivial (a large value would cause solver difficulty while small value results in low fidelity). In this work, we set $\epsilon = 1$. Table II shows that the *optimization-based approach can reduce the computational time of the simulation-based search by a factor of two to four.* We *validated* the solution of the optimization-based search (which includes smoothed control laws and time discretization) by fixing the controller parameters obtained and running high-fidelity dynamic simulations for the 230 scenarios. Such simulations are then used to compute expected power and the characteristic load with the *peak-over-load threshold* method.

Our results in indicate that, while both the exhaustive search and optimization-based search push P_{rated} to its largest value of $P_{rated} = 6\,MW$ and push w_{g_2} its smallest value of $w_{g_2} = 117.82\,rad/sec$, the settings for $w_{g_{ref}}$ and $w_{g_{25}}$ are quite different. The expected power obtained with optimization is 1% larger than that of the simulation-based search while the characteristic load of settings from optimization is about 1% lower than that of the exhaustive search. Figure 4 shows an instance for power and load profiles obtained with optimization, high fidelity simulation with optimal controller settings, and high-fidelity simulation with nominal settings. Compared with the simulation approach with nominal settings, the optimization approach clearly increases power output and the load. We also observe that the profiles obtained with optimization and high-fidelity validation match well.

Based on the optimal parameters obtained with optimization, we have derived a heuristic search. This consists of keeping both w_{g_2} and $w_{g_{25}}$ at the nominal value, and increase P_{rated} and $w_{g_{ref}}$. This heuristic is rarely mentioned in the literature. With only two degrees of freedom, we can discretize the space $\mathcal{U}$ using a finer mesh. Both $w_{g_{ref}}$ and P_{rated} are discretized with 6 points, resulting in $6^2 = 36$ combinations. The results are shown in Table III from where we can conclude that the optimal setting P_{rated} is $6\,MW$ and the optimal setting $w_{g_{ref}}$ is 127.83 rad/sec. Compared with the nominal, the optimal settings increase the power output from $2.64\,MW$ to $3.12\,MW$ (improvement of 18%) and $\hat{y}_L(\alpha_{50yr})$ from 166 MNm to 196 MNm (increase of 18%). This illustrates that the optimization-based approach can be used to identify more effective heuristic search strategies. From these results we also observe that the characteristic load is more sensitive to the changes in $w_{g_{ref}}$ than the changes in P_{rated}. For

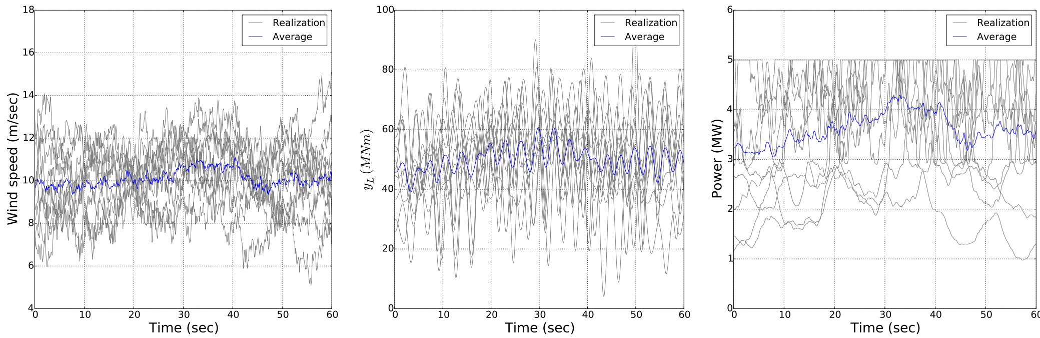

Fig. 3: **Wind, load, and power profiles for mean wind speed of** 10 **m/s.**

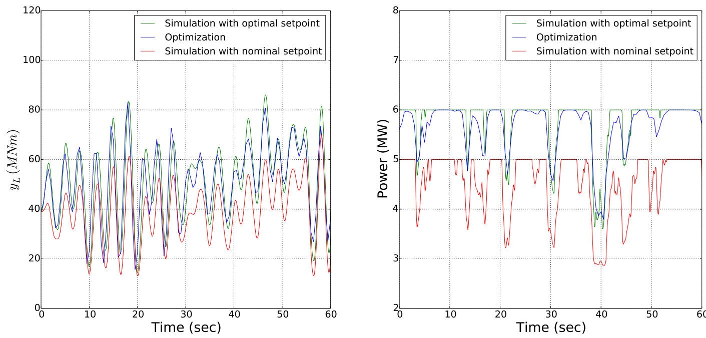

Fig. 4: **Power and load profile instances at 15** m/sec**.**

example, increasing P_{rated} from $5\,MW$ to $6\,MW$ increases the expected power from 2.64 MW to 2.94 MW at the cost of increasing $\hat{y}_L(\alpha_{50yr})$ from $166MNm$ to $175MNm$. However, increasing $w_{g_{ref}}$ from $122.91\,rad/sec$ to $147.49\,rad/sec$ only increases the expected power 2.64 MW to 2.85 MW while increases $\hat{y}_L(\alpha_{50yr})$ from $166\,MNm$ to $225\,MNm$. The 1-year characteristic load $\hat{y}_L(\alpha_{1yr})$ is notably smaller than $\hat{y}_L(\alpha_{50yr})$. If we relax the load constraint to $\hat{y}_L(\alpha_{1yr}) \leq \overline{y}_L$, then the optimal $w_{g_{ref}}$ increases to $137.66\,rad/sec$ and the optimal power increases from $3.12\,MW$ to $3.19\,MW$ (an improvement of 2%).

C. Effect of Torque Control Laws

Table IV presents results obtained with the modified torque control law. Here, we use the the proposed heuristic search to identify optimal parameters. By comparing with the results obtained with the standard control law (shown in Table III) we observe that, for almost all combinations of P_{rated} and $w_{g_{ref}}$, the change of control law has limited effect on the load but can increase the mean power output. In particular, for a load constraint $\hat{y}_L(\alpha_{50yr}) \leq \overline{y}_L = 200MNm$, the optimal power increases from 3.12 MW to 3.24 MW. Table V confirms that the parameter settings obtained with the exhaustive and optimization search for the modified control law coincide.

The different results obtained with standard and modified torque controllers highlight that the *control law has a significant effect on performance*. To understand how strong this effect can actually be, we solved an optimization instance *without control laws*. That is, the time profile of the angle $\theta(t)$ and torque $T_{gen}(t)$ are used directly as decision variables. This can be interpreted as an *MPC control strategy*. We note that this induces a large number of degrees of freedom (in this case 2,400). Consequently, a simulation-based search is no longer applicable. Table VI shows that the expected power obtained with MPC is 3.9 MW, *which represents an improvement of 25.9% relative to the optimal settings found with the use of control laws* (3.07 MW). The MPC strategy improves the annual power by 7 GWh/yr (at an electricity price of 30 USD/MWh this represents and improvement of 218,000 USD/yr. We also note that the solution times of the MPC strategy are reduced to 40 minutes, which highlight the complexity introduced by the non-smooth control laws. Although the deployment of MPC is more challenging, these results provide an idea of the potential performance. Moreover, the computational times indicate that it seems plausible that additional simplifications can be used to enable real-time MPC implementations. These results also highlight that the proposed framework can be used as a research tool to investigate the benefits of different control architectures.

V. Conclusions and Future Work

We proposed stochastic optimization formulations to identify optimal parameters for wind turbine pitch and torque controllers that maximize power and mitigate long-term extreme loads. The proposed formulation incorporates a wind turbine dynamic model, detailed pitch and torque control laws, and a rigorous procedure to estimate the probability of long-term extreme load exceedance (as described by the IEC-61400 standard). We provide evidence that the approach can significantly improve performance of nominal controller parameters. We also demonstrate that the formulation can be solved efficiently by using parallel optimization solvers.

Appendix A
Model Nomenclature

$w_r(t)$: Rotor speed [rad/sec]
$x(t)$: Tower top displacement fore-aft [m]

TABLE III: **Expected power output and characteristic loads using standard torque control law.**

$P_{rated}(MW)$	$w_{g_{ref}}(rad/sec)$	y_P (MW)	$\hat{y}_L(\alpha_{50yr})$ (MNm)	$\hat{y}_L(\alpha_{1yr})$ (MNm)
5.0	122.91	2.64	166	144
	127.83	2.78	185	159
	147.49	2.85	225	189
6.0	122.91	2.94	175	152
	127.83	3.12	196	169
	147.49	3.20	246	206

TABLE IV: **Expected power output and characteristic loads using modified torque control law.**

$P_{rated}(MW)$	$w_{g_{ref}}(rad/sec)$	y_P (MW)	$\hat{y}_L(\alpha_{50yr})$ (MNm)	$\hat{y}_L(\alpha_{1yr})$ (MNm)
5.0	122.91	2.70	166	145
	127.83	2.90	182	158
	147.49	3.22	225	191
6.0	122.91	3.00	174	152
	127.83	3.24	193	167
	147.49	3.58	244	207

TABLE V: **Results of simulation-based and optimization-based search with modified torque control law.**

Approach	$\bar{P}$ (MW)	P_{rated} (MW)	$w_{g_{ref}}$ (rad/sec)	y_P (MW)	$\hat{y}_L(\alpha_{50yr})$ (MNm)	Solution Time (hr)
Simulation		6	127.83	3.24	189	4.9
Optimization	3.2	6	128.49	3.26	196	3.3

TABLE VI: **Results of optimization without control laws (MPC).**

$\bar{P}$ (MW)	y_P (MW)	$\hat{y}_L(\alpha_{50yr})$ (MNm)	Solution Time (hr)
3.7	3.71	209	0.3
3.9	3.91	186	0.7

$v_x(t)$: Velocity of x $[m/sec]$
$\lambda(t)$: Effective tip speed ratio [-]
$C_t(t)$: Thrust coefficient [-]
$C_m(t)$: Power coefficient [-]
$F_z(t)$: Aerodynamic thrust force $[N]$
$M_z(t)$: Aerodynamic torque $[Nm]$
$V(t)$: Wind speed $[m/sec]$
$\theta(t)$: Collective pitch angle [°]
$T_{gen}(t)$: Generator torque (at high speed shaft) $[Nm]$
$y_P(t)$: Electrical power $[W]$
$y_L(t)$: Tower base bending moment (load) $[Nm]$
N_g = 97 : Gear ratio generator shaft [-]
R_r = 63 : Rotor radius $[m]$
$A = \pi R_r R_r$: Disk area $[m^2]$
J =11776047 : Total moment of inertia of the drive-train $[kgm^2]$
P_1 = 0.056 : P loss coefficient (linear) [-]
ρ = 1.225 : Air density $[kg/m^3]$
H = 90 : Tower height $[m]$
m_{Te} = 436750 : Tower equivalent modal mass $[kg]$
$\overline{y}_L$ = 200 : Maximum load threshold $[MNm]$
c_{Te} = 17782 : Tower structural damping $[kg/s]$
k_{Te} = 1810000 : Bending stiffness $[kg/s^2]$
P_{rated0} = 5 : Nominal rated power $[MW]$
$w_{r_{rated0}}$ = 1.267 : Nominal rated rotor speed $[rad/sec]$
$w_{g_{rated0}}$ = 122.91 : Nominal rated generator speed $[rad/sec]$
$w_{g_{ref0}}$ = 122.91 : Nominal reference generator speed $[rad/sec]$
$w_{g_{20}}$ = 117.82 : Nominal boundary speed $[rad/sec]$
$w_{g_{250}}$ = 121.68 : Nominal boundary speed $[rad/sec]$
V_{rated} = 11.2 : Rated wind speed $[m/sec]$
λ_{opt} = 8.22 : Optimal tip speed ratio [-]
cp_{max} = 0.51 : Peak power coefficient [-]
θ_{min} = 0 : Minimum blade pitch angle [°]
θ_{max} = 30 : Maximum blade pitch angle [°]

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