Damping Ratios are Constant

$$y(t) = AK\left[1 - \frac{1}{\beta} e^{-\xi t/\tau} \sin\left(\frac{\beta}{\tau} t + \phi\right)\right]$$

The sine function is bounded between $[-1, 1]$ so that $y(t)$ reaches peaks at times when $\sin(\beta/\tau\, t + \phi) = -1$. Since sine function is periodic, ~~the times at which this happens are~~

$$t_1$$
$$t_2 = t_1 + T$$
$$t_3 = t_1 + 2T$$
$$\vdots$$
$$t_N = t_1 + (N-1)T$$

where T is period of oscillation. The peaks are thus given by:

$$y(t_1) = AK\left[1 + e^{-\xi t_1/\tau}\right]$$

$$y(t_2) = AK\left[1 + e^{-\xi(t_1+T)/\tau}\right]$$

$$\vdots$$

$$y(t_3) = AK\left[1 + e^{-\xi(t_1+2T)/\tau}\right]$$

Now the overshoot is $a_1 = y(t_1) - y_d$

$$a_2 = y(t_2) - y_d \qquad \text{where } y_d = AK$$
$$\vdots$$
$$a_3 = y(t_3 - y_d)$$

and the ratios

$$a_2/a_1 = \frac{e^{-\xi(t_1+T)/\tau}}{e^{-\xi(t_1)/\tau}} = e^{-\xi T/\tau} \longrightarrow \underline{\text{ratios are constant}}$$

$$a_3/a_2 = \frac{e^{-\xi(t_1+2T)/\tau}}{e^{-\xi(t_1+T)/\tau}} = e^{-\xi T/\tau}$$