$\boxed{4.4}$ (a) Take Laplace transforms in (P4.3), (P4.4) to obtain, upon rearrangement:

$$x_1(s) = \frac{u(s)}{(s+k_1)} \qquad (4.11)$$

and

$$x_2(s) = \frac{k_1 x_1(s)}{(s+k_2)} \qquad (4.12)$$

Introducing (4.11) into (4.12) for x_1 gives

$$x_2(s) = \frac{k_1 u(s)}{(s+k_1)(s+k_2)}$$

or, from (P4.5),

$$\boxed{y(s) = \frac{k_1 u(s)}{(s+k_1)(s+k_2)}} \qquad (4.13)$$

(b) In vector-matrix form, the model is:

$$\dot{\underset{\sim}{x}} = \frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} -k_1 & 0 \\ k_1 & -k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

so that $\underset{\sim}{A} = \begin{bmatrix} -k_1 & 0 \\ k_1 & -k_2 \end{bmatrix}$; $\underset{\sim}{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $\underset{\sim}{c}^T = \begin{bmatrix} 0 & 1 \end{bmatrix}$.

1/30/2016

Solution Problem 2: Am I Sleepy?

1) At steady-state:

$n_{oc}^{ss} = 0$

$$\cancel{\frac{\partial C}{\partial t}} = \frac{Q}{V}(C_{in} - C^{ss}) + \cancel{\frac{G}{V} n_{oc}^{ss}}$$

$$\Rightarrow \boxed{C^{ss} = C_{in}} \quad \text{because} \quad \begin{matrix} Q > 0 \\ V > 0 \end{matrix}$$

2)

$$\frac{\partial C}{\partial t} = \frac{Q}{V}(C_{in} - C(t)) + \frac{G}{V} n_{oc}$$

in canonical form

$$\frac{V}{Q}\frac{\partial C}{\partial t} = C_{in} - C(t) + \frac{G}{Q} n_{oc}$$

using deviations $x(t) = C(t) - C^{ss}$ & $d(t) = n_{oc}(t) - n_{oc}^{ss}$

$$\frac{V}{Q}\frac{\partial x}{\partial t} = -x(t) + \frac{G}{Q} d(t)$$

$$\tau \frac{\partial x}{\partial t} = -x(t) + K d(t)$$

$$\Rightarrow \boxed{\hat{x}(s) = \frac{K}{\tau s + 1}\hat{d}(s) \quad \text{with} \quad K = \frac{G}{Q}, \quad \tau = \frac{V}{Q}}$$

3) $n_{oc}(t) = 66 \Rightarrow d(t) = 66$ (step function)
with $A = 66$

A first order response with step function yields:

$$X(t) = AK\left(1 - e^{-t/\tau}\right)$$

with
$$X(\infty) = AK = 66\left(\frac{G}{Q}\right) = 66\left(\frac{1.1 \times 10^4 \ \frac{ppm\text{-}cf}{min}}{528 \ cf/min}\right)$$

$$\Rightarrow \boxed{X(\infty) = 1{,}375 \ ppm}$$

$C(\infty) = C^{ss} + X(\infty) = 1{,}775 \ ppm$

ASHRAE is not satisfied (1,000 ppm max)

4) Attached

5) We know that
$$x(t) = AK\left(1 - e^{-t/\tau}\right)$$

$$\Rightarrow \frac{x(t)}{AK} = 1 - e^{-t/\tau}$$

$$\Rightarrow e^{-t/\tau} = 1 - \frac{x(t)}{AK}$$

$$\Rightarrow -t/\tau = \log\left(1 - \frac{x(t)}{AK}\right) \Rightarrow t = -\tau\left(\log\left(1 - \frac{x(t)}{AK}\right)\right)$$

we know $x(t) = (1000 - 400) ppm = 600 \ ppm$

$K = G/Q = 20.83 \ ppm$

$\tau = V/Q = 18.93 \ min$ $\Rightarrow t = 10.85 \ min$

$A = 6$ to reach 1,000 ppm

Consequently, students are exposed for

$$\boxed{t = 50\,min - 10.85\,min = 39\,min}$$

6) $C(\infty) = 1000\,ppm \Rightarrow x(\infty) = 600\,ppm$

$$x(\infty) = AK$$
$$= A\frac{G}{Q}$$
$$\Rightarrow Q = \frac{AG}{x(\infty)} = \frac{66\left(1.1\times10^4\,\frac{ppm\text{-}cf}{min}\right)}{600\,ppm} = 1{,}210\,cfm$$

$$\Rightarrow \boxed{Q = \frac{1210\,cfm}{66} = 18.33\ cfm/occupant}$$

Steady state is reached after

$$t = -\tau \log\left(1 - \frac{x(\infty)}{AK}\right)$$

with $A = 66$, $x(\infty) = 600\,ppm$

$$\tau = \frac{V}{Q} = \frac{10000\,cf}{1210\,cfm} = 8.26\,min$$

$$K = \frac{G}{Q} = \frac{1.1\times10^4\,\frac{ppm\text{-}cf}{min}}{1{,}210\,cfm} = 9.09\,ppm$$

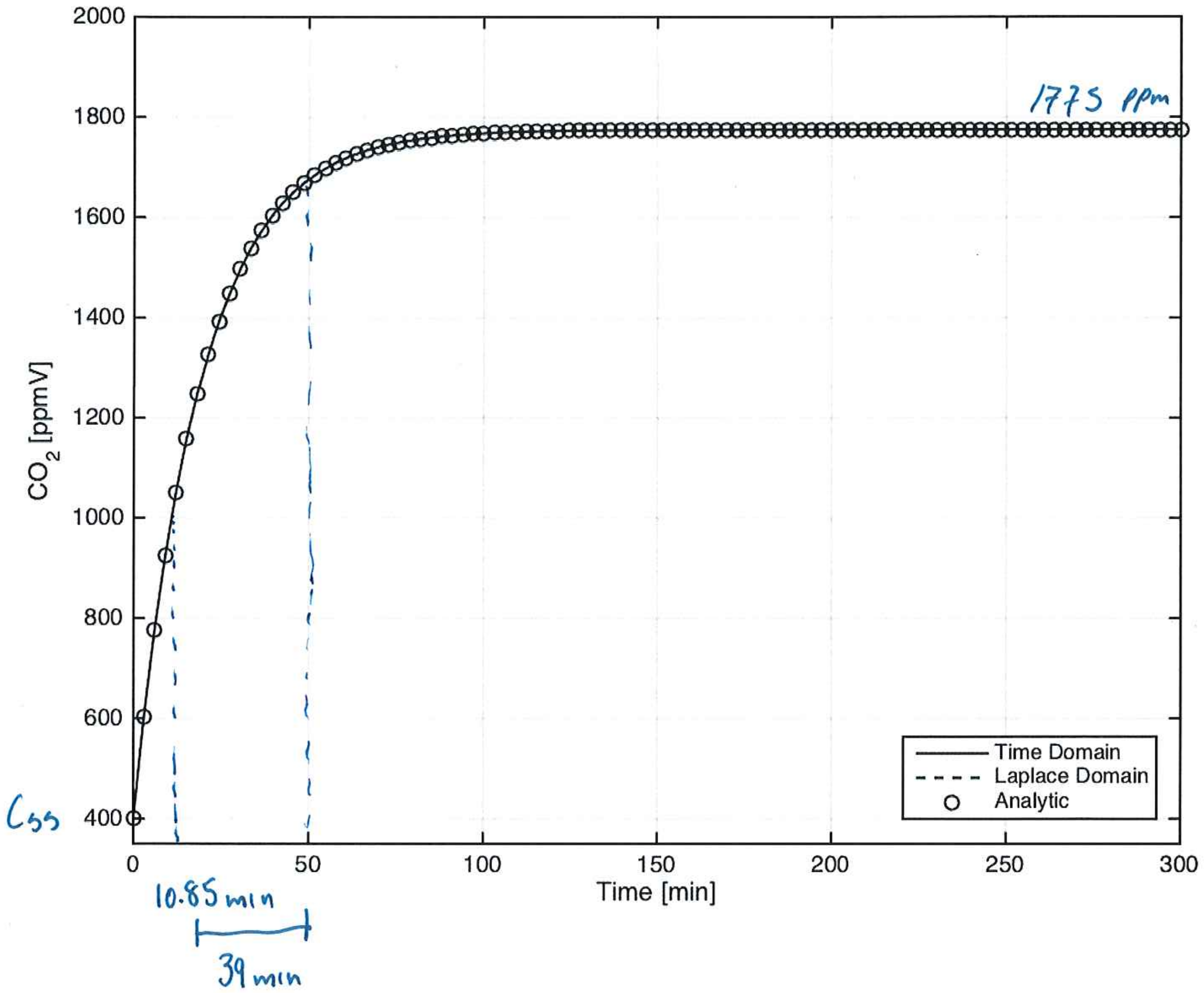
$\boxed{t = 71\,min}$ (Approximately)

Note: $X(\infty) = AK$ and thus $\log(0) = \text{Inf}$

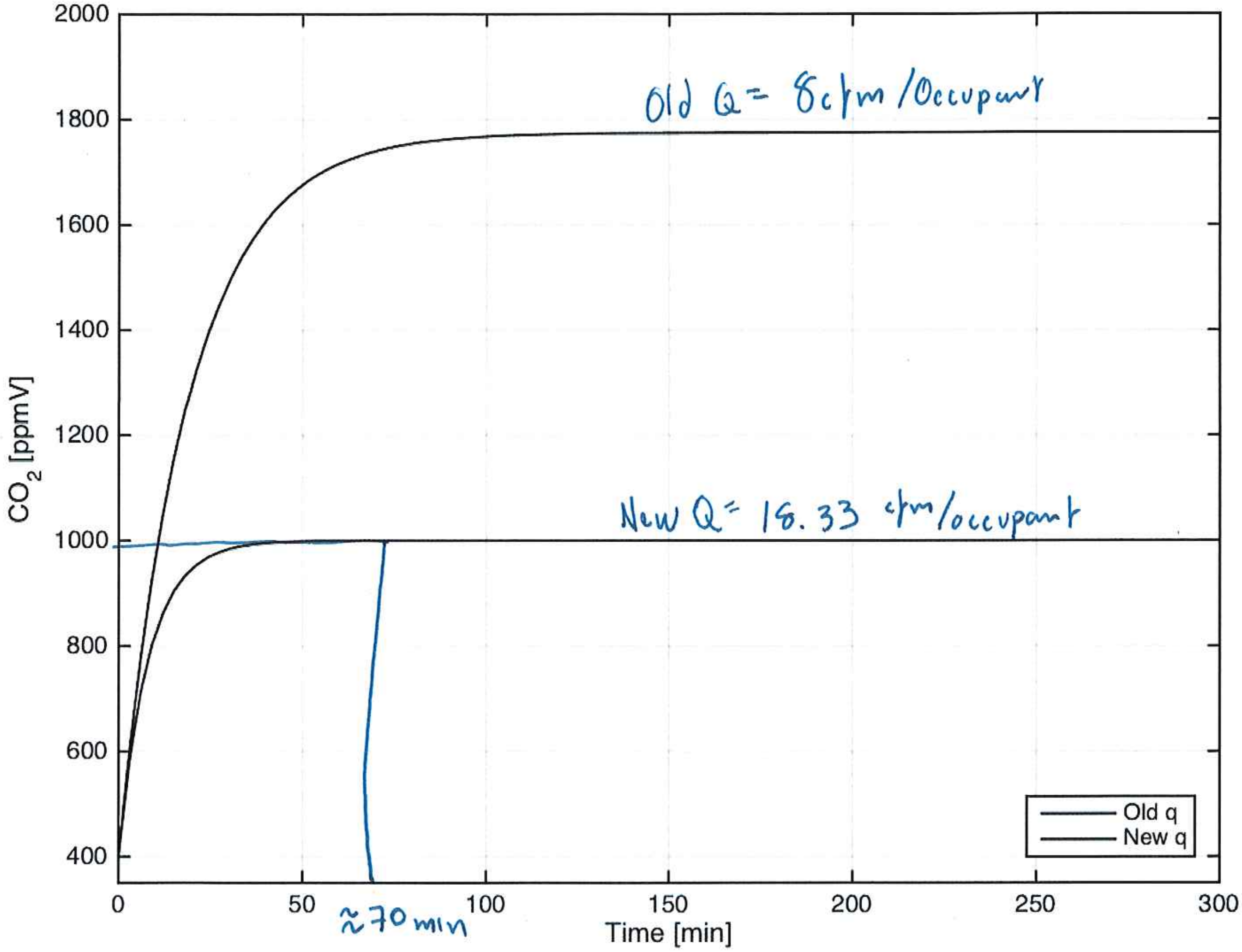
so take $X(\infty) \approx AK$

```
% hw2 problem: am I sleepy?
% victor m. zavala 2016
clc
clear all
close all hidden

 co2init = 400;          % initial conditions
       G = 1.1e4         %  ppm-cf/min  (co2 generation per occupant)
       V = 10000;        %  cf  (building volume)
    camb = 400;          % ppmV (fresh air co2 concentration)
     noc = 0;            % number of occupants
       q = 8*66;         % cfm  (air flow), ashrae standard 8 cfm/occupant
     co2 = camb;         % co2 at stedy-state

% simulate reference system
   n = 10;
time = linspace(0,300);
co2initnew = co2init;
    nocnew = noc+66;
 xinit = co2initnew-co2init;
     d = nocnew-noc;
[t,x]=ode15s('sysref',time,xinit,[],G,V,camb,q,d);

% simulate reference system with Laplace
K=G/q;
tau=V/q;

num = K;
den = [tau 1];
sys = tf(num,den);
[xx,tt]= step(sys,time); % simulates unit step
    xx = nocnew*xx; % multiply by amplitude of step

 % analytic solution
    xxx = (66*K)*(1-exp(-t./tau));

figure(1)
plot(t,co2+x,'blue')
hold on
plot(t,co2+xx,'red--')
hold on
plot(t,co2+xxx,'o')
axis([0 max(time) 350 2000])
legend('Time Domain','Laplace Domain','Analytic')
xlabel('Time [min]')
ylabel('CO_2 [ppmV]')
grid on
legend('location','southeast')

print -dpdf co2responsehw2.pdf


% simulate reference system with Laplace
q = 18.33*66;         % cfm  (air flow), ashrae standard 8 cfm/occupant
K=G/q;
tau=V/q;


 % analytic solution
    xxx = (66*K)*(1-exp(-t./tau));

figure(2)
plot(t,co2+x)
hold on
plot(t,co2+xxx)
axis([0 max(time) 350 2000])
legend('Old q','New q')
xlabel('Time [min]')
ylabel('CO_2 [ppmV]')
grid on
legend('location','southeast')

print -dpdf co2responsehw2_2.pdf
```

4), 6)



2000
1800
1600
1400
1200
1000
800
600
400
0
50
100
150
200
250
300
CO2 [ppmV]
Time [min]
Time Domain
Laplace Domain
Analytic
1775 ppm
Css
10.85 min
39 min


Old Q = 8cfm/Occupant
New Q = 18.33 cfm/occupant
≈70 min
CO2 [ppmV]
Time [min]
Old q
New q
2000
1800
1600
1400
1200
1000
800
600
400
0
50
100
150
200
250
300