$\boxed{6.9}$ Laplace transforms of the given model equations:

$$sx_1(s) = -k_1x_1(s) + u(s)$$
$$sx_2(s) = k_1x_1(s) - k_2x_2(s) \;;\quad y(s) = x_2(s)$$

Easily rearranges to give

$$\boxed{y(s) = \frac{k_1}{(s+k_1)(s+k_2)}\,u(s)} \tag{6.18}$$

(a) Given $k_1 = 5.63$, $k_2 = 12.62$, and $u(t)$ as in Fig P6.1, we have

$$y(s) = \frac{5.63}{(s+5.63)(s+12.62)}\left[\frac{10(1-e^{-5s})}{s}\right] \tag{6.19}$$

By partial fraction expansion, this becomes:

$$y(s) = \left[\frac{0.7923}{s} + \frac{0.638}{(s+12.62)} - \frac{1.4306}{(s+5.63)}\right](1-e^{-5s})$$

with the following inverse Laplace transform:

$$y(t) = \begin{cases} 0.7923 + 0.638e^{-12.62t} - 1.4306e^{-5.63t} \;; & 0<t<5 \\ 0.638\left(e^{-12.62t} - e^{-12.62(t-5)}\right) - 1.4306\left(e^{-5.63t} - e^{-5.63(t-5)}\right); & t \geq 5 \end{cases}$$

Max occurs at $t=5$, and $y(5) = 0.7923$

(b) With $u(t) = 10e^{-k_2 t}$; $t \geq 0$, we have

$$u(s) = \frac{10}{(s+k_2)}$$

and the required response is obtained from

$$y(s) = \frac{10k_1}{(s+k_1)(s+k_2)^2} = \frac{56.3}{(s+5.63)(s+12.62)^2}$$

$$= \frac{A}{(s+5.63)} + \frac{B}{(s+12.62)} + \frac{C}{(s+12.62)^2}$$

Obtain $A = 1.152$; $B = -1.152$; $C = -8.054$

and therefore

$$y(t) = 1.152e^{-5.63t} - 1.152e^{-12.62t} - 8.054te^{-12.62t}$$

(c) With the given $u(t)$, we have

$$u(s) = \frac{20}{s^2+4}$$

and now

$$y(s) = \frac{112.6}{(s+5.63)(s+12.62)} \cdot \frac{1}{(s^2+4)}$$

$$= \frac{A}{(s+5.63)} + \frac{B}{(s+12.62)} + \frac{Cs+D}{s^2+4}.$$

Obtain $A = 0.4512$; $B = -0.0986$; $C = -0.3519$;
$D = 1.2974$

~~Answers:~~

Problem 2: The Curious Case of the Bode Diagram

1. Define y as the value returned by Matlab and AR as defined in class. The relationship between the two is $y = 20\log_{10}(AR)$. Command `doc bode` has the answer.

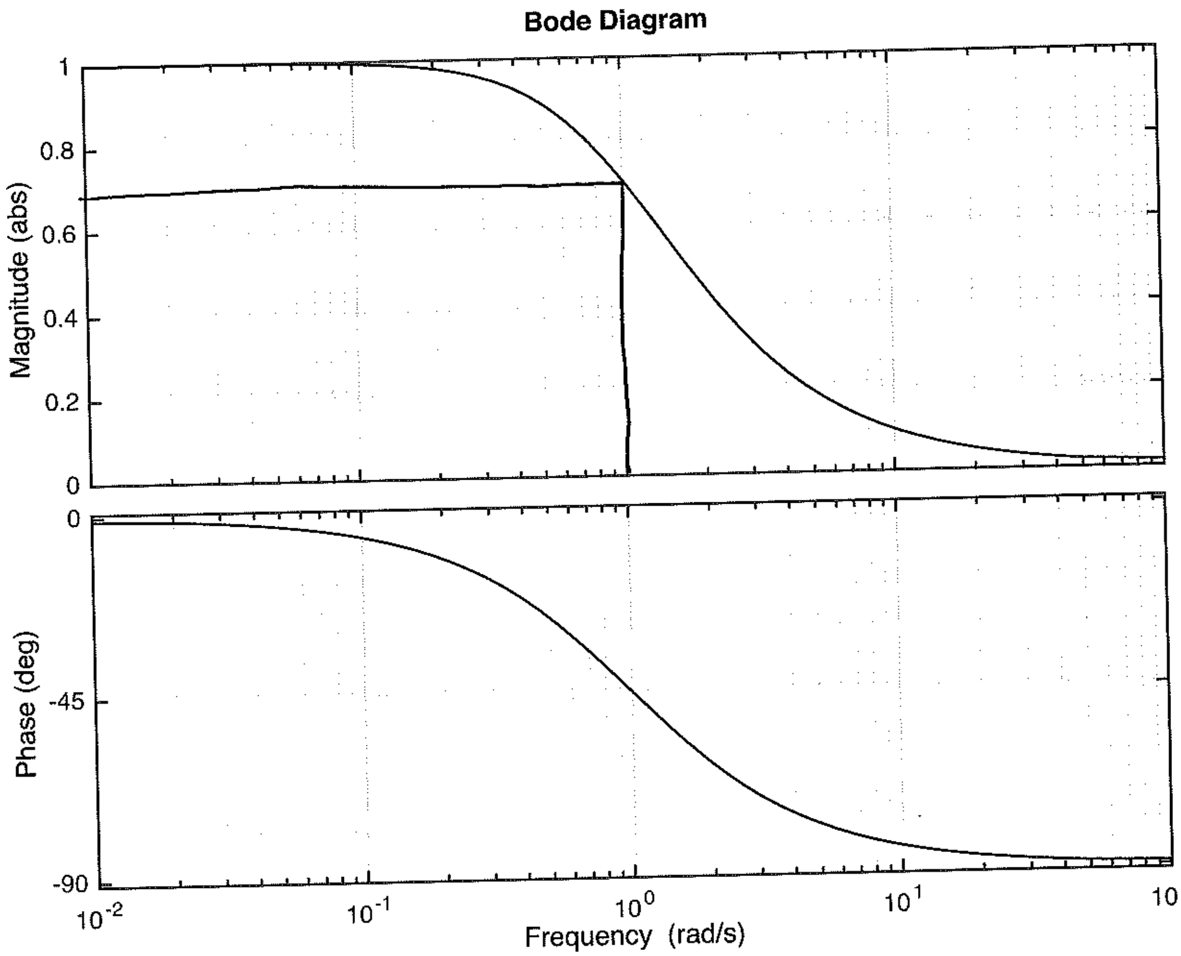
2. Command `doc bodeoptions` has the answer. Generate Bode plot with absolute values of AR using `h=bodeplot(sys)` and `setoptions(h,'MagUnits','abs')`.

3.- See attached

```
clc
clear all
close all hidden


% define another system
num = [1];
den = [1 1];
sys = tf(num,den)


% get bode diagram
figure(1)
w=[1e-1 1e0 1e1]
[mag,phase]=bode(sys,w);
h=bodeplot(sys)
grid on

print -dpdf bodedefault.pdf

% get bode diagram
figure(2)
w=[1e-1 1e0 1e1]
[mag,phase]=bode(sys,w);
h=bodeplot(sys)
setoptions(h,'MagUnits','abs')
grid on

print -dpdf bodeabs.pdf
```

$$AR = \frac{K}{\sqrt{(\tau\omega)^2+1}} \qquad @\ K, \tau = 1,\ \omega = 1$$

$$AR = \frac{1}{\sqrt{2}} \rightarrow AR = 0.7071$$



$y = 20 \log_{10}(0.7071) = -3.1$