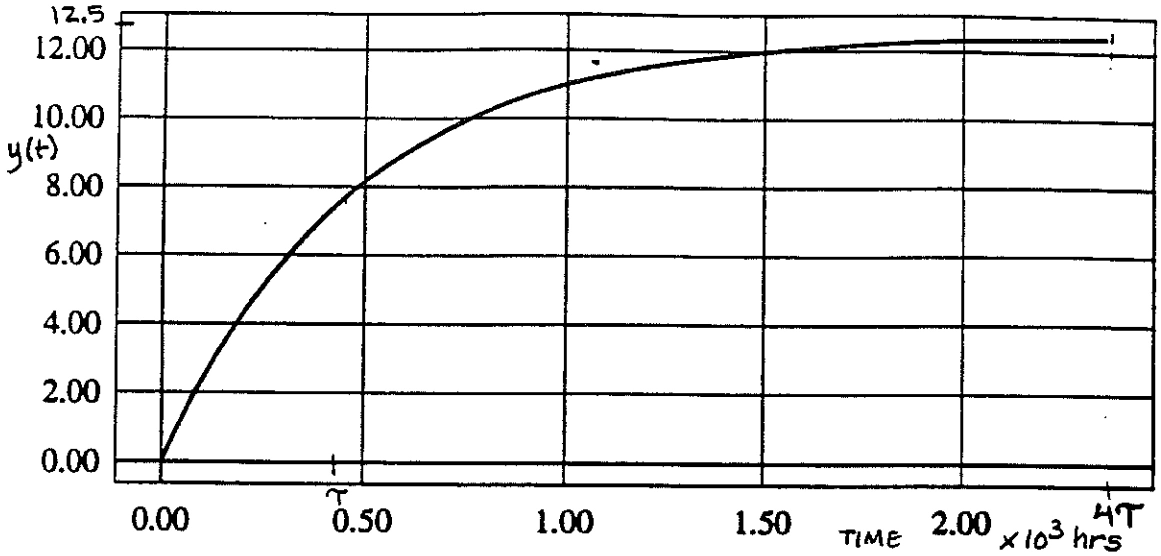

FIG S14·5

14.4 (a) consolidate inner loop first; then obtain overall closed loop transfer function as:

$$y = \psi y_d + \psi_{d_1} d_1 + \psi_{d_2} d_2$$

with

$$\psi = \frac{g_{c_1}\left(\dfrac{g_{c_2} g_2}{1 + g_{c_2} g_2 h_2}\right) g_1 g}{1 + g_{c_1}\left(\dfrac{g_{c_2} g_2}{1 + g_{c_2} g_2 h_2}\right) g_1 g h_3 h_1}$$

$$\psi_{d_1} = \frac{g_{d_1}}{1 + g g_1\left(\dfrac{g_{c_2} g_2}{1 + g_2 g_{c_2} h_2}\right) g_{c_1} h_1 h_3}$$

$$\psi_{d_2} = \frac{\left(\dfrac{g_{d_2}}{1 + g_2 g_{c_2} h_2}\right)}{1 + g g_1 \left(\dfrac{g_2 g_{c_2}}{1 + g_2 g_{c_2} h_2}\right) g_{c_1} h_1 h_3}$$

which may be further simplified, if desired.

(b) Consolidate the inner loop first; then obtain overall closed loop transfer function as

$$\psi = \frac{\dfrac{g g_c}{1 + g_K g_c}}{1 + \dfrac{g g_c}{1 + g_K g_c}}$$

which simplifies to:

$$\psi = \frac{g^* e^{-\alpha s} g_c}{1 + (g + g_K) g_c} \qquad (14\cdot10)$$

so that with $g_K = g^* - g^* e^{-\alpha s}$, ψ becomes

$$\psi = \frac{g^* g e^{-\alpha s}}{1 + g^* g_c}$$

and the characteristic equation,

$$1 + g^* g_c = 0$$

does not include the time delay element.

with $g_k = 0$, the characteristic equation is:

$$1 + g^* g_c e^{-\alpha s} = 0$$

which contains the time delay element.

$\boxed{14.5}$ If the outputs of each of the parallel processes are designated v_1 and v_2 then

$$y = g_d d + v_1 + v_2$$

with
$$v_1 = g_1 g_{c_1}(y_d - y)$$
$$v_2 = g_2 g_{c_2}(y_d - y)$$

so that

$$y = g_1 g_{c_1}(y_d - y) + g_2 g_{c_2}(y_d - y) + g_d d$$

This rearranges to give

$$y = \psi(s)\, y_d + \psi_d(s)\, d \qquad (14.11)$$

where

$$\psi = \frac{g_1 g_{c_1} + g_2 g_{c_2}}{1 + g_1 g_{c_1} + g_2 g_{c_2}} \qquad (14.12)$$

and

$$\psi_d = \frac{g_d}{1 + g_1 g_{c_1} + g_2 g_{c_2}} \qquad (14.13)$$

Now, for there to be no offset, we have the following conditions which ψ and ψ_d must satisfy.

$$\lim_{s \to 0} \psi(s) = 1 \tag{14.14a}$$

and
$$\lim_{s \to 0} \psi_d(s) = 0 \tag{14.14b}$$

(these conditions may be obtained using the final value theorem of Laplace transforms.)

Under circumstances in which

$g_{c_1} = K_{c_1}$, a pure proportional controller

$g_{c_2} = \dfrac{K_{I_2}}{s}$, a pure integral controller

Eq (14.12) becomes

$$\psi(s) = \frac{s g_1 K_{c_1} + g_2 K_{I_2}}{s + s g_1 K_{c_1} + g_2 K_{I_2}}$$

so that
$$\lim_{s \to 0} \psi(s) = \frac{g_2 K_{I_2}}{g_2 K_{I_2}} = 1$$

Similarly, Eq (14.13) becomes

$$\psi_d(s) = \frac{s g_d}{s + s g_1 K_{c_1} + g_2 K_{I_2}}$$

so that
$$\lim_{s \to 0} \psi_d(s) = 0$$

thus establishing that, as required, there will be no offsets.