Time-Shift Property

1) $\boxed{f(t-a) = L^{-1}\{\hat{f}(s)e^{-as}\}}$

2) $\boxed{\begin{aligned} L^{-1}\{\hat{f}(s-a)\} &= e^{at}L^{-1}\{\hat{f}(s)\} \\ &= e^{at}f(t) \end{aligned}}$ $\Rightarrow$ 3) $\boxed{L\{e^{at}f(t)\} = \hat{f}(s-a)}$

Example (HW3)

$$y(s) = \underbrace{\left(\frac{A_0}{s} + \frac{A_1}{s+k_1} + \frac{A_2}{s+k_2}\right)}_{\hat{f}(s)}\left(10(1-e^{-\alpha s})\right)$$

We know that

I) $f(t) = L^{-1}\{\hat{f}(s)\} = \underbrace{A_0}_{\text{because } L^{-1}\{\frac{1}{s}\}=1} + \underbrace{A_1 e^{-k_1 t}}_{\text{because } L^{-1}\{\frac{1}{s+k_1}\} = e^{-k_1}L^{-1}\{\frac{1}{s}\}} + A_2 e^{-k_2 t}$

similar argument

$\Rightarrow$ Use of 2) with $\hat{f}(s) = \frac{1}{s}$

$a = -k_1$

$\hat{f}(s-a) = \frac{1}{s-a}$

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So now we know that

$$y(s) = \hat{f}(s)10 - 10\hat{f}(s)e^{-\alpha s}$$

$\Rightarrow$

$$y(t) = L^{-1}\{y(s)\} = \underbrace{10L^{-1}\{\hat{f}(s)\}}_{\Downarrow \text{ From I)}} - 10\underbrace{L^{-1}\{\hat{f}(s)e^{-\alpha s}\}}_{\Downarrow}$$

$$= 10(A_0 + A_1e^{-k_1t} + A_2e^{-k_2t}) - 10(A_0 + A_1e^{-k_1(t-\alpha)} + A_2e^{-k_2(t-\alpha)})$$

$$\boxed{y(t) = 10A_1(e^{-k_1t} - e^{-k_1(t-\alpha)}) + 10A_2(e^{-k_2t} - e^{-k_2(t-\alpha)})}$$

which holds for $t \geq \alpha$

Example (HW3)

$$u(t) = 10e^{-k_2 t}$$

$$L\{u(t)\} = \underbrace{10\,L\{e^{-k_2 t}\}}_{3)\text{ with } f(t)=1,\ \hat{f}(s)=\frac{1}{s},\ a=-k_2} = \underline{\underline{10\left(\frac{1}{s+k_2}\right)}}$$