Optimization Example Problems

CBE470: Process Dynamics and Control

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Problem 1: An interesting observation.

Prove that the optimal solution $x^* \in \Re$ of the optimization problem

$$\min_x \quad f(x) \qquad \text{where} \quad f(x) = \frac{1}{2}\sum_{k=1}^{N}(x - d_k)^2$$

is the average of the data points $d_1, d_2, ..., d_N$. How is the optimal objective value $f(x^*)$ related to the variance of the data points?

Problem 2: Quadratic programs are fun

Derive optimality conditions and find, by hand, the solution (x_1^*, x_2^*) and the optimal objective function of the optimization problem:

$$\begin{aligned} &\min\ (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - 1)^2 \\ &\text{s.t. } x_1 + x_2 = 2. \end{aligned}$$

Derive optimality conditions and find, by hand, the solution and optimal objective value if, in addition, you impose the constraint $x_1 \geq 2$:

$$\begin{aligned} &\min\ (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - 1)^2 \\ &\text{s.t. } x_1 + x_2 = 2 \\ &\qquad x_1 \geq 2. \end{aligned}$$

Could you have guessed the solutions without using the optimality conditions? How?