Final Exam

May 6th, 2016

CBE470: Process Dynamics and Control - Spring 2016

Department of Chemical and Biological Engineering, University of Wisconsin-Madison

http://zavalab.engr.wisc.edu/teaching/cbe470spring2016

Instructions

- You may use notes, books, lecture slides, electronic devices. Work alone.
- Problem 1: 50 points.
- Clearly explain how you arrived at an answer. If you just state an answer you will not get full credit.

Problem 1: Last one and we're outta here!

Assume that you have two identical classrooms that exchange energy by heat conduction through the wall. The coupled system is described by the following set of differential equations

$$\frac{dT_1(t)}{dt} = q_1(t)(T_{in} - T_1(t)) - \alpha(T_1(t) - T_2(t)) + G_1$$
$$\frac{dT_2(t)}{dt} = q_2(t)(T_{in} - T_2(t)) - \alpha(T_2(t) - T_1(t)) + G_2$$

where G_1 and G_2 are heat generation rates by the occupants in the rooms (assumed constant). The wall heat transfer coefficient is $\alpha > 0$ and the inlet air temperature is T_{in} (assumed constant). The states are the room temperatures $T_1(t)$ and $T_2(t)$. The controls are the air flow rates (per unit of volume) $q_1(t)$ and $q_2(t)$.

1. (20 pts) Linearize the system using the steady-state reference point $(q_1^{ss}, q_2^{ss}, T_1^{ss}, T_2^{ss})$ and assume that $q_1^{ss} = q_2^{ss} = q$ and $T_1^{ss} = T_2^{ss} = T$. Using your linear representation $\dot{x} = Ax + Bu$ prove that the open-loop system can *never be open-loop unstable*.

2. (20 pts) Using your linear representation $\dot{x} = Ax + Bu$ find the gain matrix K for the control law $u = Kx$ such that the closed-loop system $\dot{x} = Ax + BKx$ evolves as as the perfectly decoupled system:

$$x_1(t) = x_1(0)e^{-t}$$
$$x_2(t) = x_2(0)e^{-t}.$$

3. (10 pts) The energy cost required to ventilate the rooms is given by $\frac{1}{2}\beta(q_1^2 + q_2^2)$ where $\beta > 0$ is a cost coefficient. We want to find the optimal steady-state values $q_1^{ss}, q_2^{ss}, T_1^{ss}, T_2^{ss}$ that minimize the energy cost and that keep the room temperatures above a certain threshold T_{min}. This can be done by solving the optimization problem:

$$\begin{aligned} \min \quad & \frac{1}{2}\beta(q_1^2 + q_2^2) \\ \text{s.t.} \quad & 0 = q_1(T_{in} - T_1) - \alpha(T_1 - T_2) + G_1 \\ & 0 = q_2(T_{in} - T_2) - \alpha(T_2 - T_1) + G_2 \\ & T_1 \geq T_{min} \\ & T_2 \geq T_{min}. \end{aligned}$$

   where q_1, q_2, T_1, T_2 are the problem variables and $\alpha, \beta, G_1, G_2, T_{in}, T_{min}$ are fixed parameters. Derive the optimality conditions for this problem.