HW#2

Due: February 3rd, 2017

CBE470: Process Dynamics and Control - Spring 2017

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`http://zavalab.engr.wisc.edu/teaching/cbe470spring2017`

Problem 1: Textbook Problem 4.4 (a)

Problem 2: Am I sleepy?

The dynamics of the CO_2 concentration in a classroom can be described by the first-order ordinary differential equation:

$$\frac{dC(t)}{dt} = \frac{Q}{V}(C_{in} - C(t)) + \frac{G}{V}n_{oc}(t), \tag{1}$$

where $C(t)$ is the CO_2 concentration in $[ppm]$, t is time in $[min]$, Q is the airflow rate in $[cfm]$=$[ft^3/min]$, $C_{in} = 400$ is the CO_2 concentration in the fresh airflow in $[ppm]$, $V = 10,000$ is the classroom volume in $[cf]$=$[ft^3]$, $G = 1.1 \times 10^4$ is the CO_2 generation rate per occupant in $[ppm - cf/min]$, and $n_{oc}(t)$ is the number of occupants at time t. All the units in the model are consistent. The American Society of Heating, Refrigerating, and Air-Conditioning Engineers (ASHRAE) recommends that the CO_2 concentration should be maintained below 1,000 ppm in order to prevent drowsiness. To satisfy this, a control engineer in charge of the ventilation system of the building has estimated that an airflow rate of 8 cfm *per occupant* is sufficient to stay below the CO_2 threshold. Unfortunately, the classroom vent cannot be controlled in real-time based on the actual number of occupants and thus the engineer keeps a fixed ventilation rate at all times that equals the capacity of the room. The capacity is assumed to be 66 occupants and thus $Q = 8 \times 66 = 528\, cfm$.

Address the following:

1. What is the steady-state value of the CO_2 concentration when the classroom is empty $n_{oc}^{ss} = 0$?
2. Use the steady-state as reference to define $x(t) = C(t) - C^{ss}$ and $d(t) = n_{oc}(t) - n_{oc}^{ss}$ and obtain a transfer function between the CO_2 concentration and the number of occupants of the form:

   $$\hat{x}(s) = \frac{K}{\tau\, s + 1}\hat{d}(s). \tag{2}$$

   Express K and τ as a function of Q, V and G.
3. Assume now that 66 students show up to class. What would be the new steady-state value for the CO_2 concentration? Does the airflow estimate of the control engineer satisfy the ASHRAE recommendation at steady-state?
4. Use the Matlab function `step` to compute the response of $x(t)$ to a step function in $n_{oc}(t)$ from 0 to 66 over a time interval of $t \in [0, 300]$ minutes. Provide a plot of t v.s. $C(t)$.
5. If the lecture lasts 50 minutes, for how long will the students be exposed to a CO_2 level higher than the ASHRAE recommendation?
6. What is the flowrate per occupant that yields a steady-state CO_2 concentration of 1,000 ppm with 66 occupants? Using this flow rate, how long would it take for the classroom to reach the steady-state if starting with an empty classroom?