$\boxed{14.13}$ (a) The root locus diagram is shown in FIG S14.6 from where we observe that when $K_c = 2$, the roots are $0 \pm 2j$, so that

(i) the closed loop system is stable for $K_c > 2$;
(ii) when on the verge of instability, the system oscillates with a frequency $\omega = 2$ rad/sec.

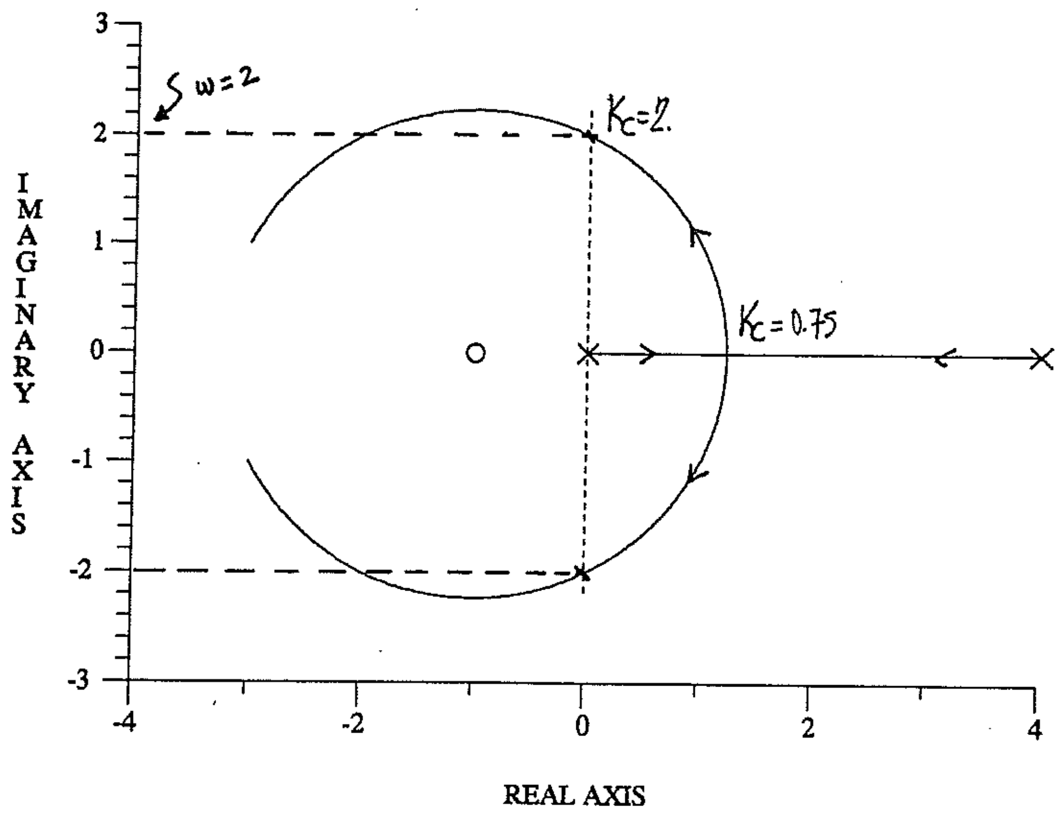


FIG S14.6

It is important to note that stability requires $K_c > 2$; for $K_c < 2$ observe that the roots are in the RHP.

(b) The characteristic equation is:

$$1 + K_c\left(1+\frac{1}{s}\right)\left(\frac{2}{s-4}\right) = 0 \qquad (14.49)$$

or $$s^2 + (2K_c - 4)s + 2K_c = 0 \qquad (14.50)$$

a quadratic which indicates that for stability,

$$2K_c - 4 > 0$$

or $$2K_c > 4$$

or $$K_c > 2$$

The critical value $K_c = 2$ substituted into (14.50) gives

$$s^2 + 4 = 0$$

with the pair of purely imaginary roots

$$s = \pm 2j$$

so that the frequency of sustained oscillation is 2 rad/sec, confirming the results of part (a).

$\boxed{14.14}$ Solution not provided. Exercise requires anticipatory input from students which is later confirmed (or refuted!) by actual simulation.

CHAPTER 15

$\boxed{15.1}$ The closed loop transfer function relation for this control system is given by

$$y = \frac{\left(\frac{-0.025}{1.5s+1}\right) K_c\left(1+\frac{1}{\tau_I s}\right)}{1+\left(\frac{-0.025}{1.5s+1}\right) K_c\left(1+\frac{1}{\tau_I s}\right)} \quad y_d$$

or $$y = \frac{-0.025 K_c(\tau_I s+1).}{1.5\tau_I s^2 + (1-0.025K_c)\tau_I s - 0.025K_c} \quad y_d. \qquad (15.1)$$

(a) The closed loop poles are determined from the quadratic

$$1.5\tau_I s^2 + (1-0.025K_c)\tau_I s - 0.025K_c = 0$$

or, dividing through by $1.5\,\tau_I$ (provided $\tau_I \neq 0$),

$$s^2 + \frac{(1-0.025K_c)}{1.5} s - \frac{0.025K_c}{1.5\tau_I} = 0 \qquad (15.2)$$

These poles are to be assigned to the same location as those of the roots of

$$s^2 + s + 1 = 0 \qquad (P15.2)$$

By comparing Eq (15.2) with (P15.2), observe that appropriate pole assignment requires that

$$\frac{1-0.025K_c}{1.5} = 1 \quad \Rightarrow \quad K_c = -20 \, ;$$

and

$$-\frac{0.025K_c}{1.5\,\tau_I} = 1 \quad \Rightarrow \quad 1/\tau_I = 3, \text{ or } \tau_I = 1/3$$

With these parameter values, the closed loop zero is located at $s = -1/\tau_I$ or $s = -3$. (See Eq (15.1) above.) Note the negative value taken by K_c; this is because the process gain is also negative.

(b) The performance of this PI controller on the process, for a step change of -10 ppm in set point, is shown below in FIG S15.1

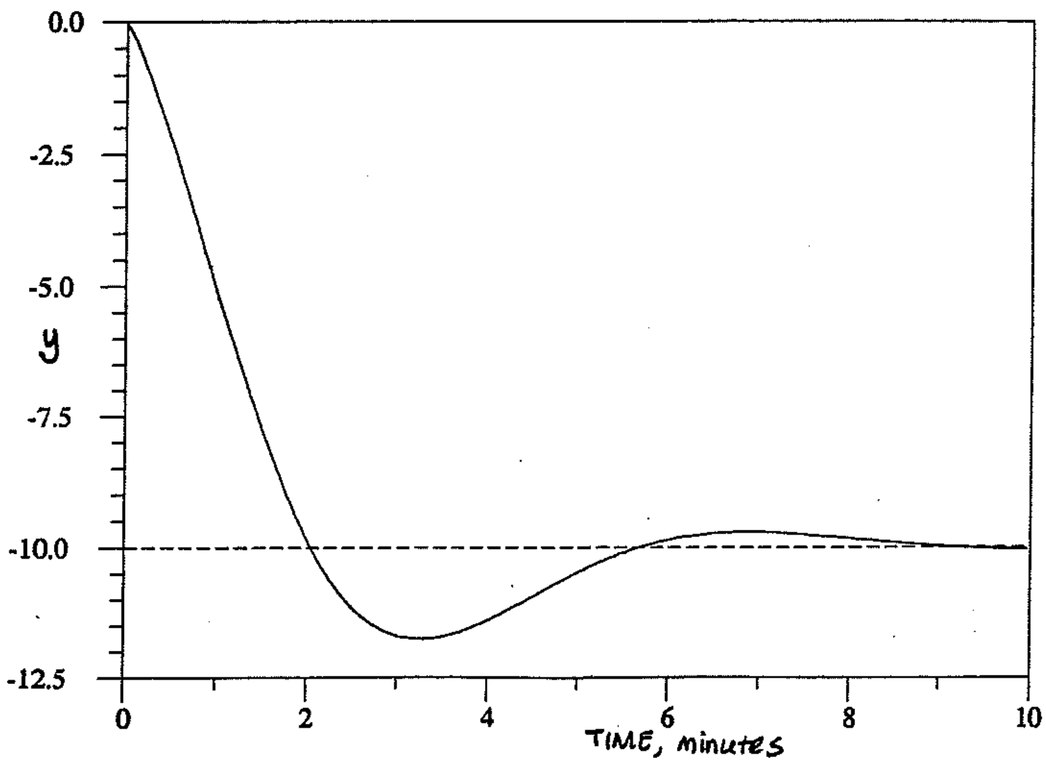


FIG S15.1

(c) A block diagram for the "real" process including the controller designed in (a) and implemented in (b) is shown below in FIG S15.2

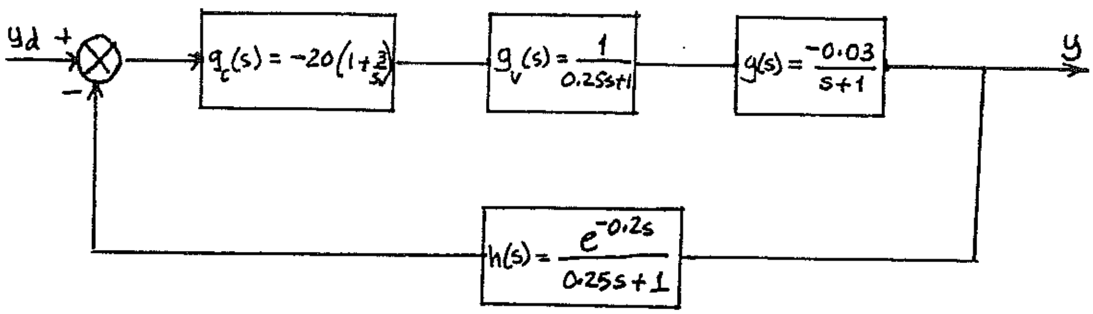


FIG S15.2

The closed loop response to the same −10 set-point change is shown below in FIG S15.3. The difference in the performance is significant enough to warrant a redesign: the "real" process response is too oscillatory.

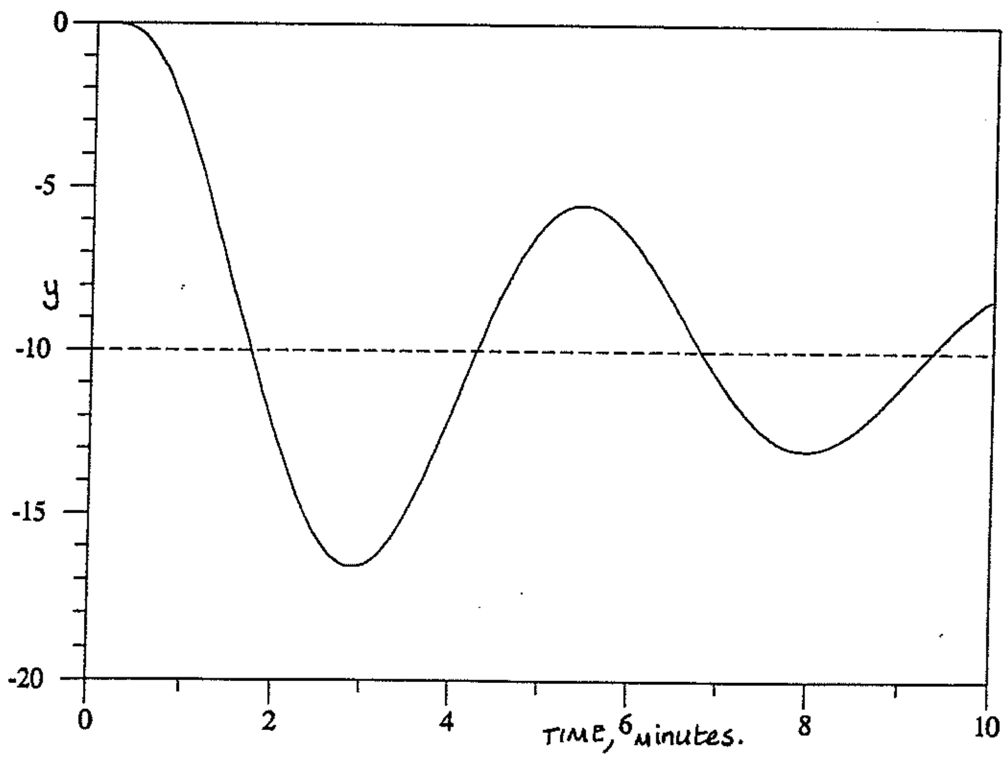
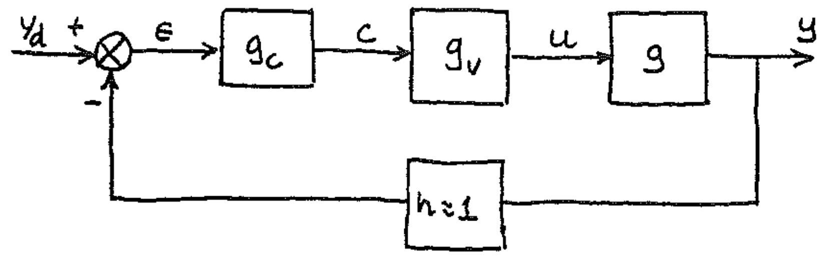


FIG S15.3

$\boxed{15.5}$ (a) A block diagram for the control system is shown below:



In this case, the OLTF g_L is given by

$$g_L = g_c g_v g = K_c\left(\frac{1.2}{0.8s+1}\right)\left(\frac{0.1(-2s+1)}{s(5s+1)}\right)$$

or $$g_L = \frac{0.12\,K_c(-2s+1)}{4s^3+5.8s^2+s} \qquad (15.9)$$

A Bode plot for this g_L is shown in FIG S15.9. From here we obtain

$$\omega_c = 0.24\ ;\quad AR_c = 0.32$$

$$\Rightarrow\quad K_{c_u} = \frac{1}{AR_c} = 3.125$$

Using a gain margin of 1.7 gives

$$\boxed{K_c^{GM} = \frac{K_{c_u}}{1.7} = 1.84} \qquad (15.10)$$

Returning to the Bode diagram, a phase margin of 30° gives $AR_{PM} = 0.67$ so that now

$$\boxed{K_c^{PM} = \frac{1}{0.67} = 1.5} \qquad (15.11)$$

Observe that the PM approach gives a more conservative controller.

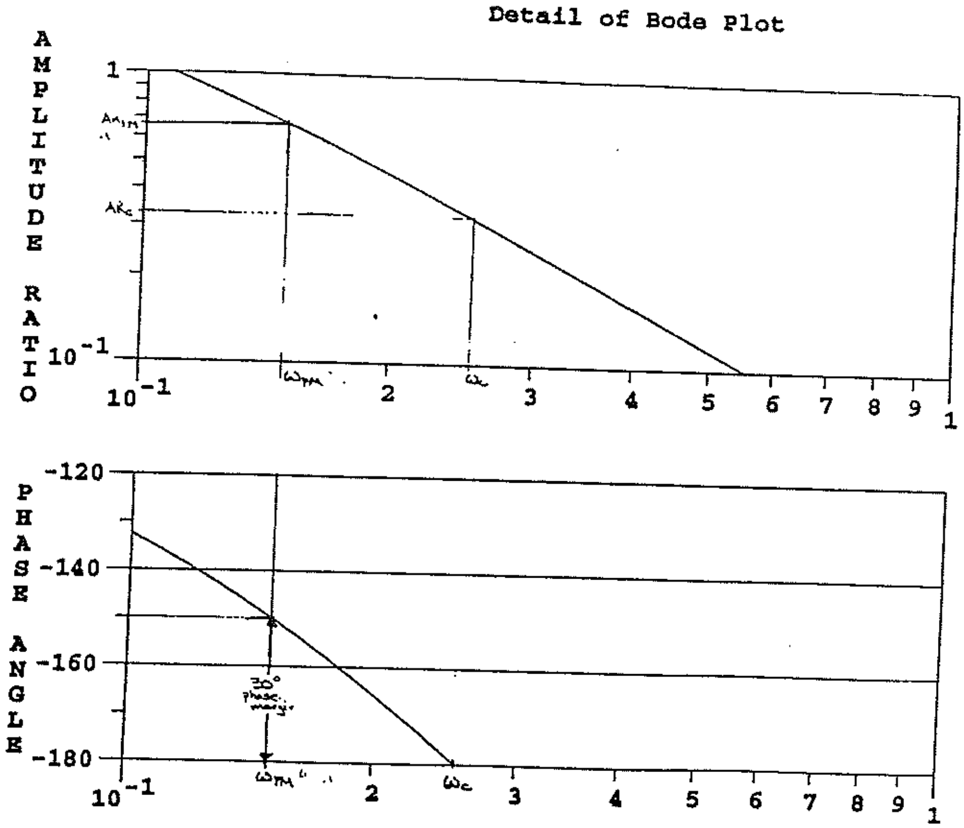


FIG S15.9

(b) The simulations showing the performance of each controller are in FIG S15.10a and FIG S15.10b. The controllers are essentially similar, the one based on GM considerations being somewhat more agressive. The overall performance obtained from the controllers are comparable. Note that there is no offset, even though we have implemented pure proportional controllers. This is because the process has an inherent integrator element. (see process transfer function).

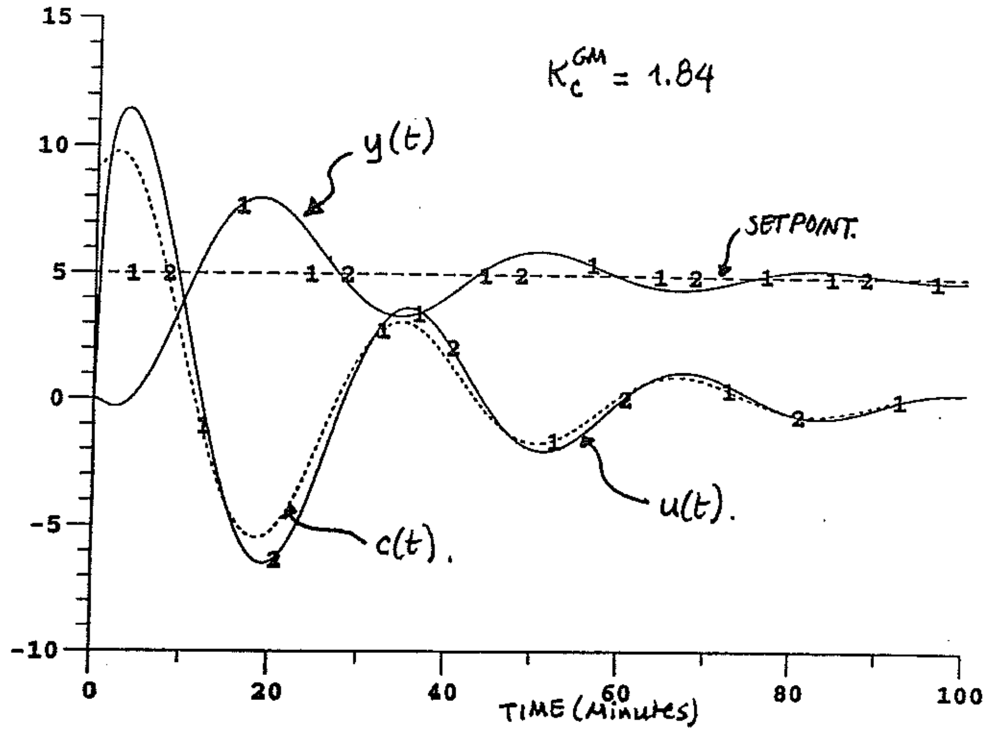


FIG S15.10a

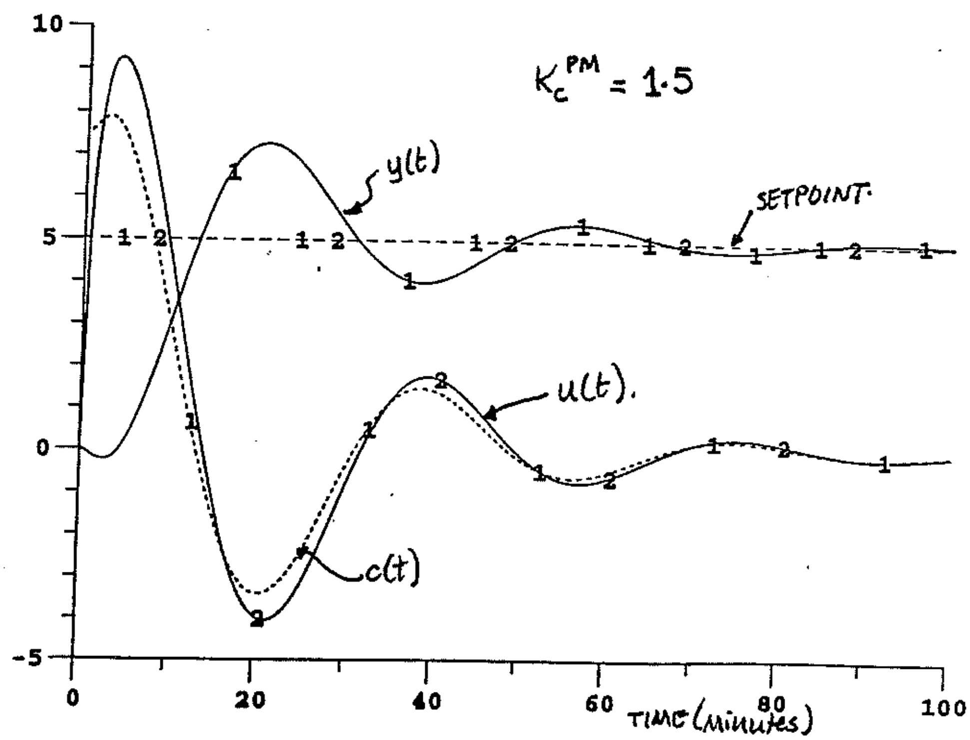


FIG S15.10b