Midterm # II

March 17th, 2016

CBE470: Process Dynamics and Control - Spring 2016

Department of Chemical and Biological Engineering, University of Wisconsin-Madison

http://zavalab.engr.wisc.edu/teaching/cbe470spring2016

Instructions

- You may use notes, books, lecture slides, electronic devices.
- Use electronic devices at your own risk. If we catch you communicating with anyone you will automatically get a grade of **zero**.
- Clearly explain how you arrived at an answer. If you just state an answer you will not get full credit.
- The points assigned to each problem are:
  - Problem 1: 20 points
  - Problem 2: 40 points
  - Problem 3: 20 points
- Start solving the easier problems first, then move to the harder ones.

Problem 1: Block Diagrams

Assume that you have a system of the form:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & 0 \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} + \begin{bmatrix} g_1^d(s) \\ g_2^d(s) \end{bmatrix} d(s)$$

Draw the corresponding *closed-loop* diagram showing the interactions between the control loops.

Problem 2: Am I Sleepy and Cold?

The dynamics of CO_2 concentration and air temperature in the CBE470 classroom can be described by the following system of nonlinear ODEs:

$$\frac{dC(t)}{dt} = \frac{Q(t)}{V}(C_{in}(t) - C(t)) + n_{oc}(t)G_c$$
$$\frac{dT(t)}{dt} = \frac{Q(t)}{V}(T_{in}(t) - T(t)) + n_{oc}(t)\frac{G_T}{\rho C_p}$$

where G_c, G_T, ρ, C_p, V are parameters. The model variables are: CO_2 concentration $C(t)$, air temperature $T(t)$, inlet air concentration $C_{in}(t)$, inlet air temperature $T_{in}(t)$, air flow rate $Q(t)$, and number of occupants $n_{oc}(t)$.

1. (10 pts) If your objective is to control $C(t)$ and $T(t)$, use your engineering judgement to select which variables to use as controls and which ones to leave as disturbances. Explain your selection.
2. (10 pts) Based on your selection, what types of sensors and actuators would you need?
3. (20 pts) Based on your selection, derive a linearized state-space representation for the system using deviation variables around the steady-state ($C^{ss}, T^{ss}, C_{in}^{ss}, T_{in}^{ss}, n_{oc}^{ss}, Q^{ss}$).

Problem 3: Controller Design

Consider the system

$$g(s) = \frac{K(\xi s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

1. (10 pts) Show that the direct synthesis controller for a desired reference trajectory of the form

$$q(s) = \frac{1}{\tau_r s + 1},$$

   is given by a PID controller of the form:

$$g_c(s) = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\left(\frac{1}{\xi s + 1}\right)$$

   and express K_c, τ_I, τ_D as a function of the parameters of $g(s)$ and $q(s)$.

2. (10 pts) Show that the closed-loop system is not affected by the zero of $g(s)$.