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Important Laplace Transforms

⇒ Basic

$$L\{1\} = 1/s \quad , \quad L\{kf(t)\} = kL\{f(t)\}, \quad L^{-1}\{\hat{f}(s-a)\} = e^{at}L^{-1}\{\hat{f}(s)\}$$

$$L\left\{\frac{\partial x(t)}{\partial t}\right\} = s\hat{x}(s) - x(0)$$

$$L\{f(t-a)\} = e^{-as}\hat{f}(s) \quad \Rightarrow \quad L^{-1}\{e^{-as}\hat{f}(s)\} = f(t-a)$$

⇒ Derived

$$L^{-1}\left\{\frac{k}{s+1/\tau}\right\} = kL^{-1}\{\hat{f}(s-a)\} = ke^{at}L^{-1}\{\hat{f}(s)\}$$

with $\hat{f}(s) = \frac{1}{s}$, $a = -\frac{1}{\tau}$

$$= ke^{at}$$
$$= ke^{-t/\tau}$$

⇒ $$\tau\frac{\partial y}{\partial t} + y = Ku(t) \qquad y(0) = 0$$

$$\hat{y}(s) = \frac{K}{\tau s+1}\hat{u}(s)$$

$$\boxed{\tau\frac{\partial y}{\partial t} + y = Ku(t) \qquad \hat{y}(s) = \frac{K}{(\tau s+1)}\hat{u}(s) + \frac{\tau y(0)}{(\tau s+1)}}$$

$y(0) \neq 0$

also note that $$\tau\frac{\partial y}{\partial t} + y = K_1u(t) + K_2d(t)$$

$$\Rightarrow \hat{y}(s) = \frac{K_1}{\tau s+1}\hat{u}(s) + \frac{K_2\hat{d}(s)}{\tau s+1} + \frac{\tau y(0)}{\tau s+1}$$

⇒ ~~Some useful formulas~~

~~LK~~

⇒ More derived

$$\boxed{\hat{y}(s) = \frac{K}{\tau s+1}\hat{u}(s) \quad \text{with } \hat{u}(s) = \frac{1}{s}}$$

$$\hat{y}(s) = \frac{K}{(\tau s+1)}\frac{1}{s}$$

$$\hat{y}(s) = -\frac{K\tau}{\tau s+1} + \frac{K}{s}$$

see appendix textbook

$$\boxed{y(t) = K(1-e^{-t/\tau})}$$

⇒ $$\boxed{\hat{y}(s) = \frac{K}{\tau s+1}e^{-as}\hat{u}(s) \quad \text{with } \hat{u}(s) = 1/s}$$

$\cancel{L^{-1}\{\hat{f}(s-a)}$ $\quad L^{-1}\{\hat{f}(s-a)\} = e^{at}L^{-1}\{f(t)\}$

note that $f(t) = K(1-e^{-t/\tau})$

and thus

$\cancel{L^{-1}\{\hat{y}(s)\} =}$ $$\boxed{y(t) = K(1-e^{-t/\tau})e^{at}}$$

More generally

$$\hat{y}(s) = g(s)\,\hat{u}(s)$$

$$\boxed{\hat{y}(s) = g(s)\,e^{-as}\,\hat{u}(s)}$$

$$\boxed{y(t) = L^{-1}\{g(s)\,\hat{u}(s)\}\,e^{at}}$$

Example: $g(s)\,\hat{u}(s) = \dfrac{K}{\tau s+1}\,\dfrac{1}{s}$

$$L^{-1}\{g(s)\,\hat{u}(s)\} = K(1-e^{-t/\tau})$$

so that

$$y(t) = K(1-e^{-t/\tau})\,e^{at}$$