Proof of final value theorem 2/1/2016

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$

where $F(s) = \mathcal{L}\{f(t)\}$

We know that

$$\cancel{\lim_{s\to 0} F(s) = \lim_{s\to 0} \int_0^\infty e^{-st} f(t)\,dt} \qquad \mathcal{L}\left\{\frac{\partial f(t)}{\partial t}\right\} = sF(s) - f(0)$$

Consequently,

$$\lim_{s\to 0} \int_0^\infty \frac{\partial f(t)}{\partial t} e^{-st}\,dt = \lim_{s\to 0} \left(sF(s) - f(0)\right) \qquad 1)$$

we ~~know~~ also know that

$$\lim_{s\to 0} \int_0^\infty \frac{\partial f(t)}{\partial t} e^{-st}\,dt = \int_0^\infty \lim_{s\to 0} e^{-st} \frac{\partial f(t)}{dt}\,dt$$

$$= \int_0^\infty df(t) = f(\infty) - f(0) \qquad 2)$$

Consequently, from 1) & 2)

$$\lim_{s\to 0} \left(sF(s) - \cancel{f(0)}\right) = f(\infty) - \cancel{f(0)}$$

$$\Rightarrow \boxed{\lim_{s\to 0} sF(s) = f(\infty)}$$

$$\Rightarrow \underline{\lim_{s\to 0} sF(s) = \lim_{t\to\infty} f(t)}$$

2/1/2016

Proof of initial value theorem

$$\boxed{\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)}$$

We know that:

$$\lim_{s \to \infty} \int_0^\infty \frac{\partial f(t)}{\partial t} e^{-st} dt = \lim_{s \to \infty} \left(sF(s) - f(0)\right) \qquad 1)$$

and

$$\lim_{s \to \infty} \int_0^\infty \frac{\partial f(t)}{\partial t} e^{-st} dt = 0 \qquad 2)$$

Consequently, combining 1) & 2):

$$\lim_{s \to \infty} sF(s) - f(0) = 0$$

$$\Rightarrow \boxed{\lim_{s \to \infty} sF(s) = f(0)}$$

$$\Rightarrow \underline{\lim_{s \to \infty} sF(s) = \lim_{t \to 0} f(t)}$$ Q.E.D.