Eulor Idantities

$$\cos(wt) = \left(e^{jwt} + e^{-jwt}\right)/2$$

$$\sin(wt) = \left(e^{jwt} - e^{-jwt}\right)/2j$$

$z = x + jy$ $\quad$ $x = Re(z)$ $\quad$ $j = \sqrt{-1}$ $\quad$ $j^2 = -1$

$r = |z|^2$ $\quad$ $y = Im(z)$

Complament

$$\bar{z} = x - jy$$

Proparties: $\quad Re(z) = \dfrac{z + \bar{z}}{2}$ $\quad\Big|\quad$ $Im(z) = \dfrac{z - \bar{z}}{2j}$

$$z\bar{z} = (x+jy)(x-jy) = \cancel{x^2 - 2j^2xy + j^2y^2}$$

$$x^2 + jxy - jxy + y^2 \cancel{= x^2 + 2xy + y^2}$$

$$\underline{z\bar{z} = |z|^2 = x^2 + y^2}$$

Polar

$z = r < \theta$ $\quad$ $\theta = \arg(z)$ phase $\quad$ $\theta = \angle z$

$r =$ amplituda

exponantial polar

$$z = re^{j\theta}$$

$$p\sin(\theta) + q\cos(\theta) = \rho\sin(\theta + \psi)$$

$$\rho = \sqrt{p^2 + q^2} \qquad \psi = \tan^{-1}\{p/q\}$$

Proof of UPR !

$$\boxed{y(t)=\int_0^\infty g(\tau)\,u(t-\tau)\,d\tau}$$

$$UPR = \int_0^\infty h(\tau)\sin(\omega(t-\tau))\,d\tau$$

$$= \frac{1}{2j}\int_0^\infty h(\tau)\left(e^{j\omega(t-\tau)}-e^{-j\omega(t-\tau)}\right)d\tau$$

$$= \frac{1}{2j}e^{j\omega t}\underbrace{\int_0^\infty h(\tau)e^{-j\omega\tau}\,d\tau}_{H(+j\omega)} - \frac{1}{2j}e^{-j\omega t}\underbrace{\int_0^\infty h(\tau)e^{j\omega\tau}\,d\tau}_{H(-j\omega)}$$

$$= \frac{1}{2j}e^{j\omega t}H(+j\omega) - \frac{1}{2j}e^{-j\omega t}H(-j\omega)$$

$$= \frac{1}{2j}e^{j\omega t}(Re+jIm) - \frac{1}{2j}e^{-j\omega t}(Re-jIm)$$

$$= \frac{Re}{2j}e^{j\omega t} + \frac{j}{2j}Im\,e^{j\omega t} - \frac{1}{2j}Re\,e^{-j\omega t} + \frac{j}{2j}Im\,e^{-j\omega t}$$

$$= \frac{Re}{2j}\left(e^{j\omega t}-e^{j\omega t}\right) + \frac{Im}{2}\left(e^{j\omega t}+e^{-j\omega t}\right)$$

$$= Re\sin(\omega t) + Im\cos(\omega t)$$

$$= |H(j\omega)|\sin(\omega t+\phi) = AR(\sin(\omega t+\phi))$$

$$\phi = \angle H(j\omega)$$