



Introduction to Scheduling of Mixed Batch/Continuous Processes

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Background

- Scheduling
- Mixed batch/continuous processes
- Resource-task network (RTN)

General formulation extension

- Discrete-time model
- Continuous-time model



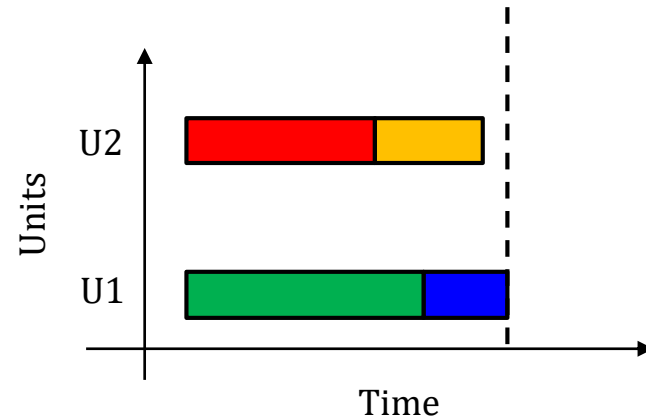
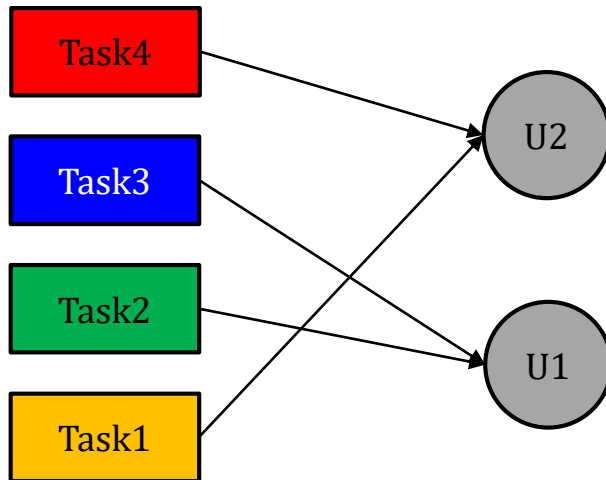
Background: Scheduling



Scheduling

- “It is the allocation of limited resources to tasks over time.
- Resources:
Equipment, raw materials and manpower, etc.
- Tasks:
Production tasks or transportation tasks, etc.
- Goal: optimization of one or more objectives
 - Maximize profit or minimize cost
 - Minimize makespan

Michael Pinedo, 1998



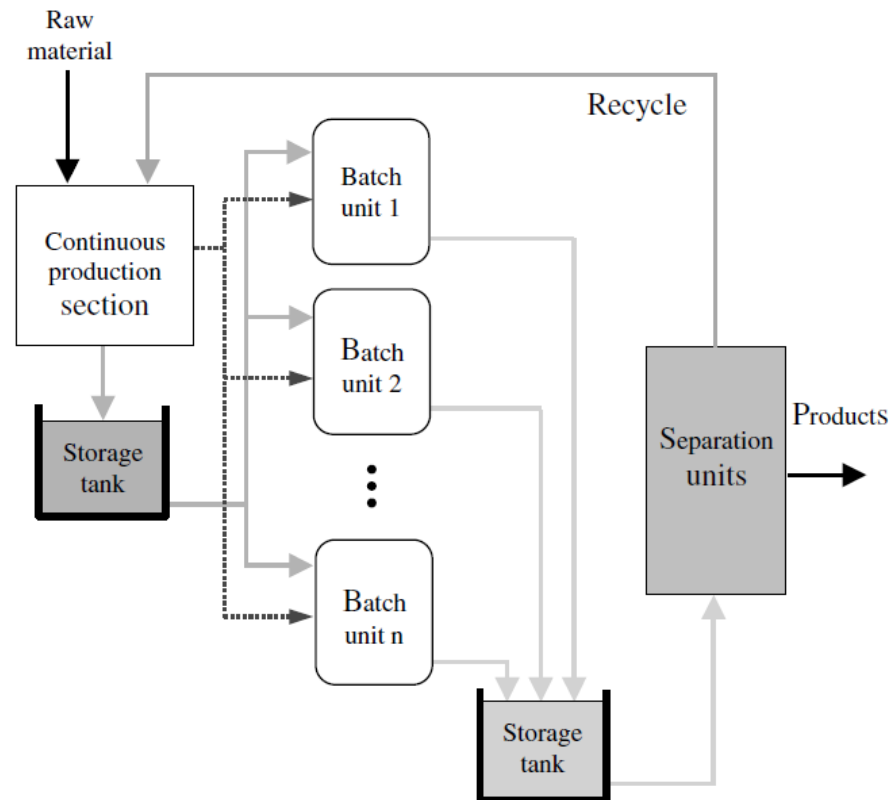


Background: Mixed Batch/Continuous Processes



Important to Industry

- Most facilities consist of multiple types of processes
- Batch processes: fermentation and blending, etc.
- Continuous processes: drying and packing, etc.
- Example: Chemical Production processes, fast moving consumer goods





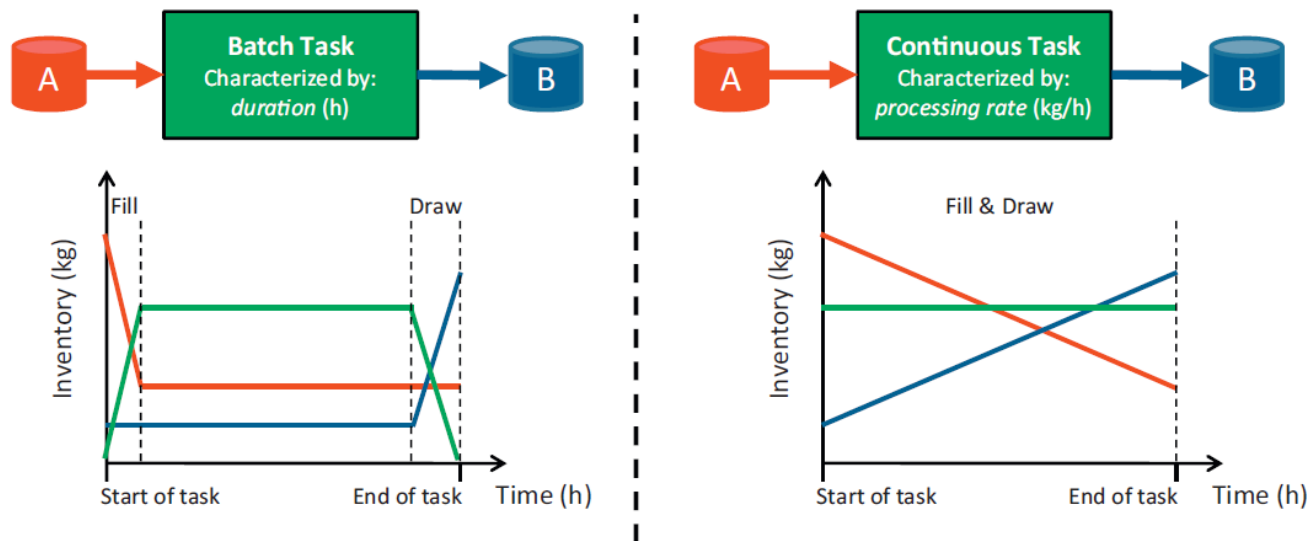
Background: Mixed Batch/Continuous Processes



From Modeling Perspective

- Main differences in capacity restrictions, processing time and effect to inventory level

Process	Capacity restrictions	Processing time
Batch	Bounded batch size	Fixed or dependent on batch size
Continuous	Bounded process rate (e.g. kg/h)	Typically explicitly not constrained





Background: Resource-task Network



Brief introduction to Resource-task Network (RTN)

- Components: resources and tasks
- Key variables and constraints

Indices:

r : resources
 i : tasks
 t : time points
 θ : shift time (time relative to the start of a task)

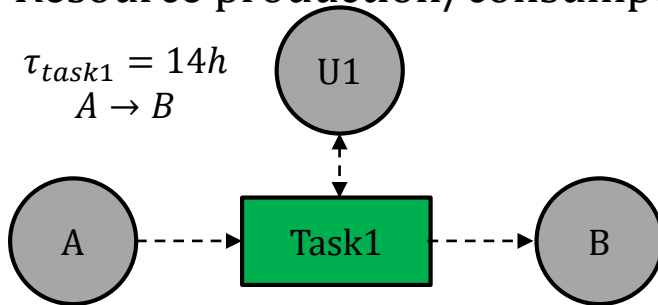
Variables:

$R_{r,t}$: excess resource
 $N_{i,t}$: assignment (binary)
 $\xi_{i,t}$: extent

Parameters:

$\mu_{i,r,\theta}$: discrete extent coefficient
 $\nu_{i,r,\theta}$: continuous extent coefficient
 τ_i : processing time

- Resource production/consumption: $\mu_{i,r,\theta}N_{i,t-\theta} + \nu_{i,r,\theta}\xi_{i,t-\theta}$



$$\begin{aligned} \nu_{task1,A,0} &= -1, \nu_{task1,B,14} = 1 \\ \mu_{task1,U1,0} &= -1, \mu_{task1,U1,14} = 1 \\ t = 0: N_{task1,0} &= 1, \xi_{task1,0} = 5 \\ &A: 0 \times 1 + (-1) \times 5 = -5 \\ &U1: (-1) \times 1 + 0 \times 5 = -1 \\ t = 14: B: &0 \times 1 + 1 \times 5 = 5 \\ &U1: 1 \times 1 + 0 \times 5 = 1 \end{aligned}$$

- Excess resource balance constraint

$$R_{r,t} = R_{r,t-1} + \sum_{i \in I_r} \sum_{\theta=0}^{\tau_i} (\mu_{i,r,\theta}N_{i,t-\theta} + \nu_{i,r,\theta}\xi_{i,t-\theta})$$

- Excess resource capacity constraint
- Operational constraint



Extension for Continuous Process



Key questions

- Batch size \rightarrow process rate?
- Account for variable process time?
- Account for inventory level that is filled/drew gradually?

Modification to key constraints of RTN

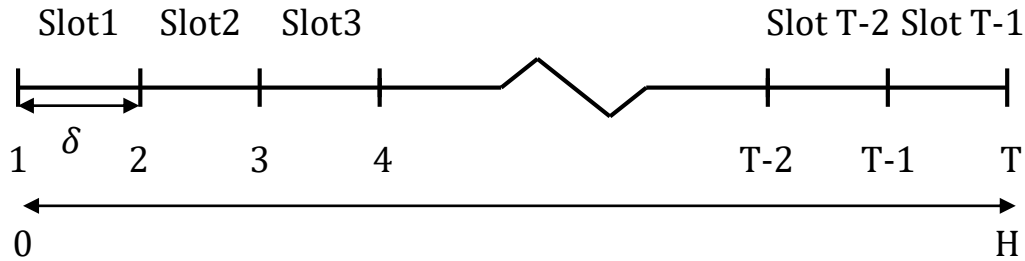
- Excess resource balance/capacity constraints & operational constraints
- Difference in continuous/discrete time representation (time-grid-base)



Discrete-time Model

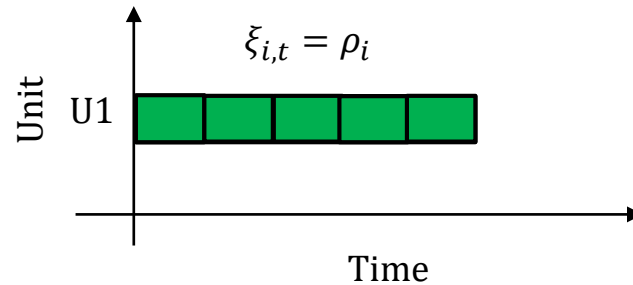
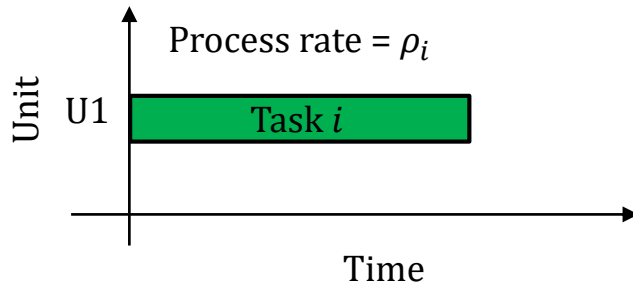


Discrete time representation



Key Modification

- Continuous process task \rightarrow batch tasks executed consecutively



- Resource production/consumption
 - Multi-extent tasks $\xi_{i,t-\theta} \rightarrow \xi_{i,m,t-\theta}$ ($m \in \mathbf{M}$: task extents)
 - $\mu_{i,r,\theta} N_{i,t-\theta} + \nu_{i,r,\theta} \xi_{i,t} \rightarrow \mu_{i,r,\theta} N_{i,t-\theta} + \sum_m \nu_{i,m,r,\theta} \xi_{i,m,t}$
- Excess resource balance constraint

$$R_{r,t} = R_{r,t-1} + \sum_{i \in I_r} \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} N_{i,t-\theta} + \sum_{i \in I_r} \sum_{m \in \mathbf{M}} \sum_{\theta=0}^{\tau_i} \nu_{i,m,r,\theta} \xi_{i,m,t-\theta}$$



Discrete-time Model

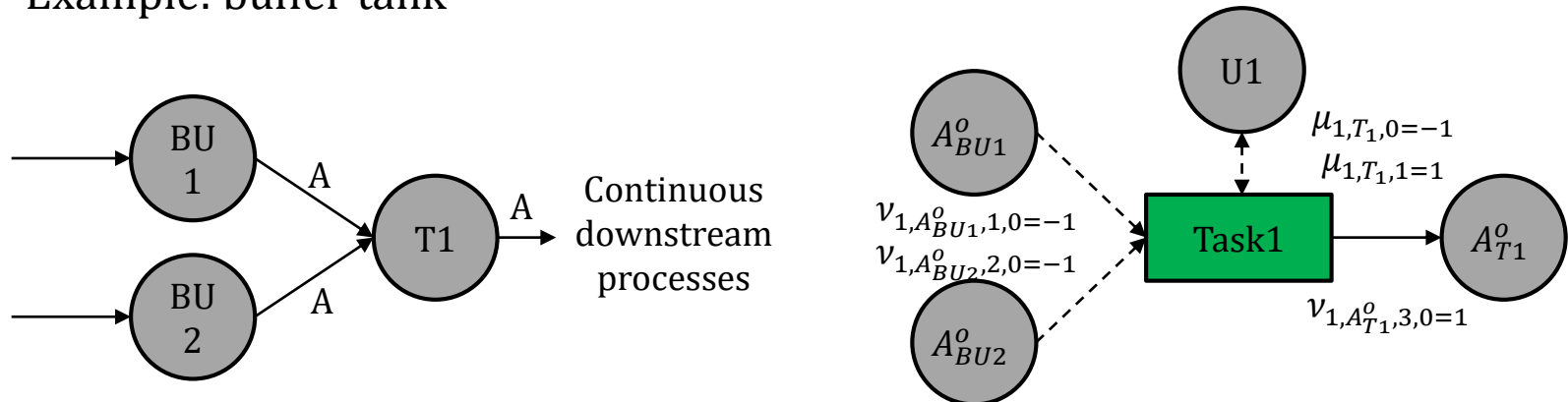


Key Modification

- Resource production/consumption
 - Multi-extent tasks $\xi_{i,t,t-\theta} \rightarrow \xi_{i,t,m,t-\theta}$ (m : task extents)
 - $\mu_{i,r,\theta}N_{i,t-\theta} + \nu_{i,r,\theta}\xi_{i,t} \rightarrow \mu_{i,r,\theta}N_{i,t-\theta} + \sum_m \nu_{i,r,\theta}\xi_{i,m,t}$
- Excess resource balance constraint

$$R_{r,t} = R_{r,t-1} + \sum_{i \in I_r} \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} N_{i,t-\theta} + \sum_{i \in I_r} \sum_{m \in M} \sum_{\theta=0}^{\tau_i} \nu_{i,r,m,\theta} \xi_{i,m,t-\theta}$$

- Applied to tasks with multiple inlet/outlet flows with independent flow rate
Example: buffer tank





Discrete-time Model



Key Modification

- Excess resource capacity constraint

- Original form: $R_{r,t}^{\min} \leq R_{r,t} \leq R_{r,t}^{\max}$
- Allow variable resource limit $R_{r,t}^{\min}$ and $R_{r,t}^{\max}$

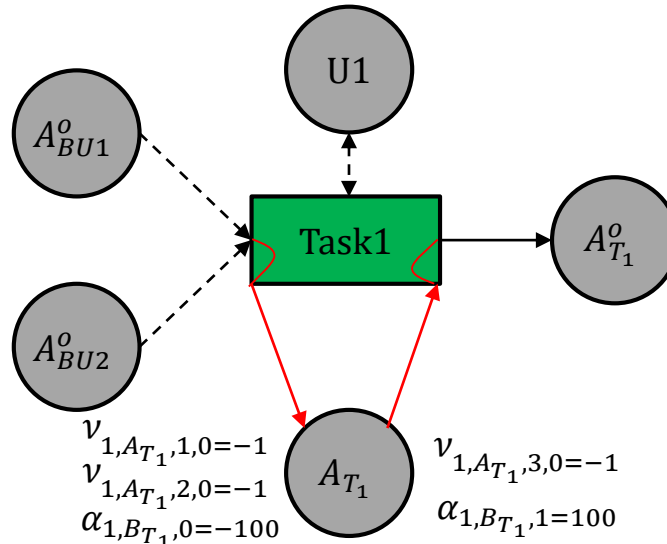
$$R_{r,t}^{\max} = R_{r,t-1}^{\max} + \sum_{i \in I_r} \sum_{\theta=0}^{\tau_i} \alpha_{i,r,\theta} N_{i,t-\theta} + \sum_{i \in I_r} \sum_{m \in M} \sum_{\theta=0}^{\tau_i} \beta_{i,r,m,\theta} \xi_{i,m,t-\theta}$$

Parameters:

$\alpha_{i,r,\theta}$: discrete resource limit coefficient

$\beta_{i,r,m,\theta}$: continuous resource limit coefficient

- Applied to storage tank under no-mixing assumption



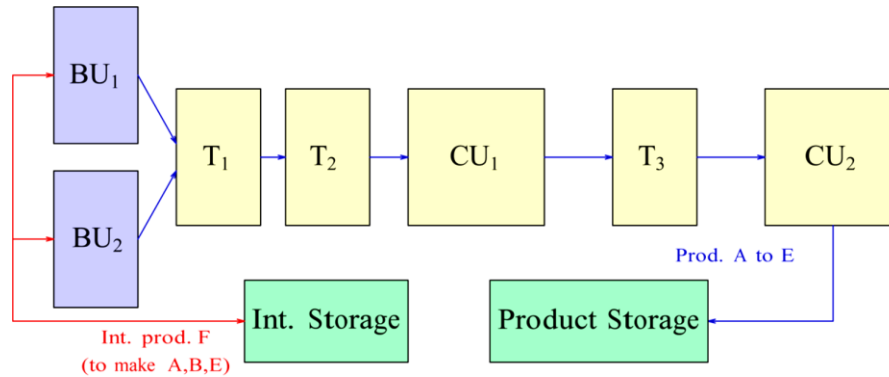


Discrete-time Model



Case study

- Reaction network



- Considering continuous tasks CU_1 and CU_2
 - Processing capacity of CU_2 is consumable
 - Transaction time losses need to be considered for tasks CU_1 and CU_2
 - Maximize profit
- Model Statistics

Obj	Gap (%)	Total Var.	Discrete Var.	Solver	CPU (s)
563	2.6	134148	4380	Gurobi 5.5.0	600

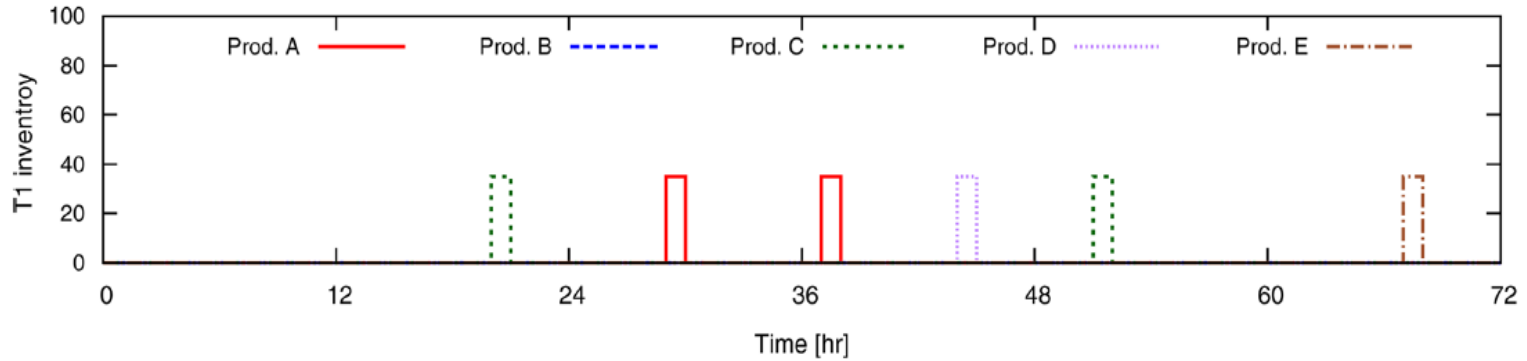
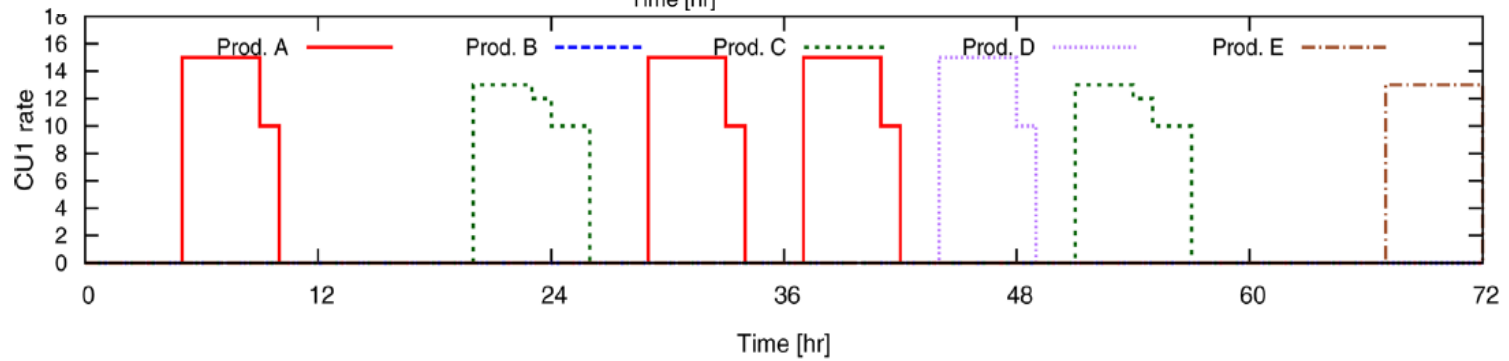
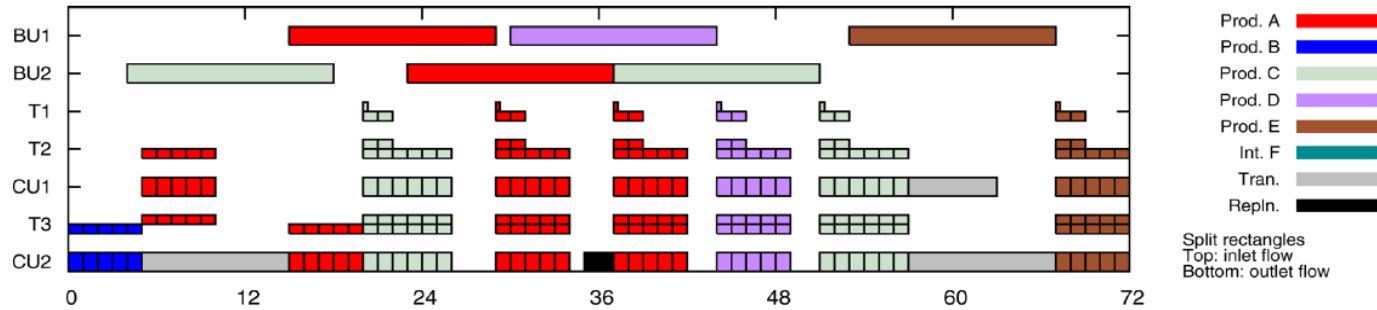


Discrete-time Model



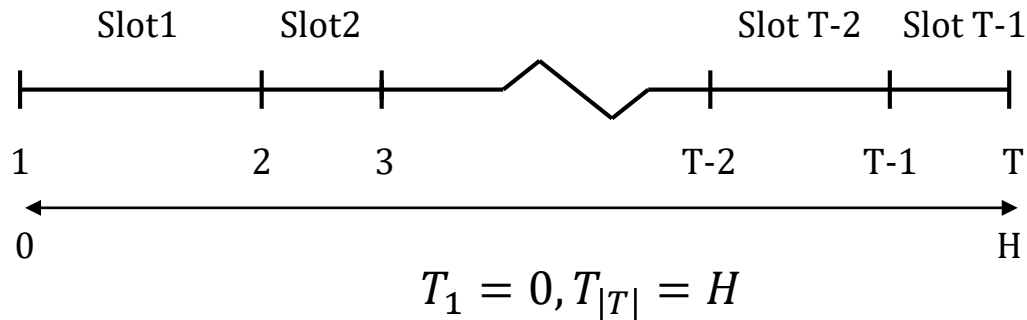
Case study

Optimal Schedule





Continuous time representation



Modified RTN representation

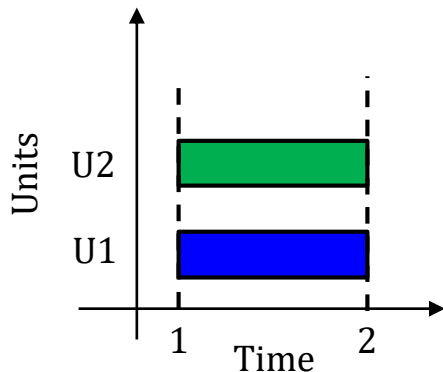
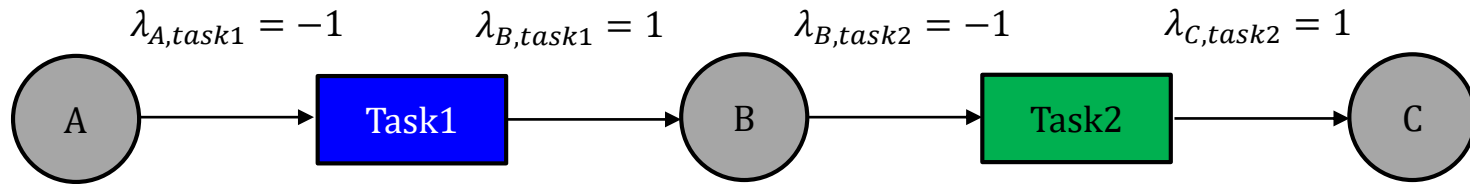
- Timing constraint
 - $N_{i,t} \rightarrow N_{i,t,t'}, \xi_{i,t} \rightarrow \xi_{i,t,t'}$
 - Batch tasks: $T_{t'} - T_t \geq \alpha_i N_{i,t,t'} + \beta_i \xi_{i,t,t'}$
 - Continuous tasks: $T_{t+1} - T_t \geq \frac{\xi_{i,t,t'}}{\rho_i^{\min}}$
- Resource production/consumption
 - Batch tasks: $\mu_{i,r} N_{i,t,t'} + \nu_{i,r} \xi_{i,t,t'}, \mu_{i,r} N_{i,t',t} + \nu_{i,r} \xi_{i,t',t}$
 - Continuous tasks:
 Instead of use term $\nu_{i,r} \xi_{i,t,t'}$, we use $\lambda_{r,i} \xi_{i,t,t+1}$
 $\mu_{i,r} N_{i,t,t+1} + \lambda_{r,i} \xi_{i,t,t+1}$



Modified RTN representation

- Excess resource balance constraints

$$\begin{aligned}
 R_{r,t} &= R_{r,t-1} + \sum_{i \in I^b} \left(\sum_{t' > t}^{t' \leq t + \delta^{\max}} \mu_{i,r} N_{i,t,t'} + \nu_{i,r} \xi_{i,t,t'} + \sum_{t' \geq t - \delta^{\max}}^{t' < t} \mu_{i,r} N_{i,t',t} + \nu_{i,r} \xi_{i,t',t} \right) \\
 &+ \sum_{i \in I^c} (\mu_{i,r} N_{i,t,t+1} + \mu_{i,r} N_{i,t-1,t} + \lambda_{r,i} \xi_{i,t-1,t})
 \end{aligned}$$



$$\begin{aligned}
 N_{task1,1,2} &= N_{task2,1,2} = 1 \\
 \xi_{task1,1,2} &= \xi_{task2,1,2} = 100 \\
 R_{A,1} &= 100, R_{B,1} = 0, R_{C,1} = 0 \\
 R_{A,2} &= R_{A,1} + (-1) \times 100 = 0 \\
 R_{B,2} &= R_{B,1} + (-1) \times 100 + 1 \times 100 = 0 \\
 R_{C,2} &= R_{C,1} + 1 \times 100 = 100
 \end{aligned}$$



Modified RTN representation

- Excess resource balance constraint
 - Batch tasks: $V_i^{\min} N_{i,t,t'} \leq \xi_{i,t,t'} \leq V_i^{\max} N_{i,t,t'}$
 - Continuous tasks: $\delta^{\min} \rho_i^{\min} N_{i,t,t'} \leq \xi_{i,t,t'} \leq H \rho_i^{\max} N_{i,t,t'}$



Major References



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Thank you!