Multiperiod Blend Scheduling Problem: Model Formulations and Solution Strategies

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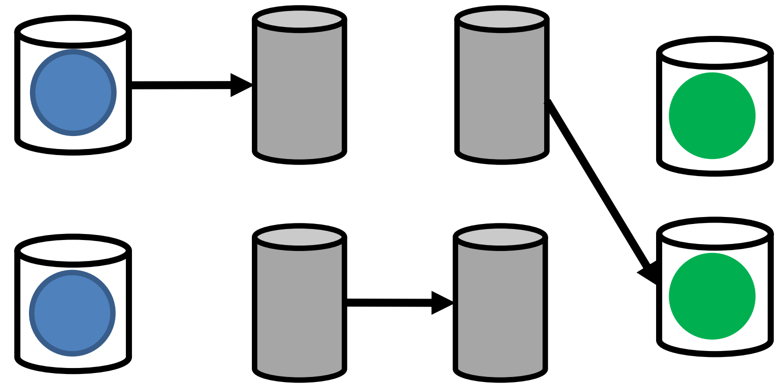
System Seminar

Nov 17th , 2017

Introduction

What is blending

Multiple *streams* with different *properties* are sent to *pools* to produce blend product, the product must satisfy certain property *specifications*



Multiperiod blending

- Steam availability/product demand vary over time
- Pools are not dedicated to certain product
- Property specifications are enforced only for the outlet flow from pools
- No simultaneous feeding/withdrawing for pools
- Properties of mixture follow linear mixing rule

Industrial practice example

Crude oil blending:

Streams – Crudes (from Saudi Arabia, North Sea…)

Properties – Sulfur, specific gravity...

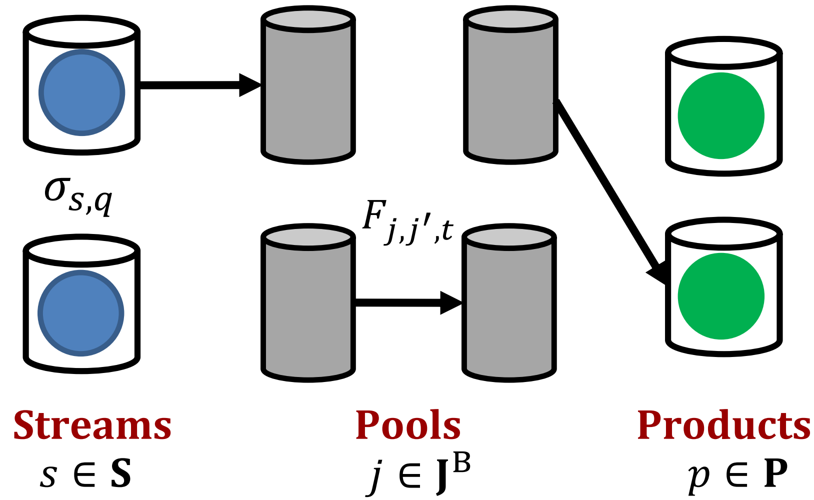
Pools – Charging tanks

Multimillion dollar benefits per year for medium-sized refinery (Kelly et al., 2003)

Problem Statement

Given

- Streams $s \in \mathbf{S}$
- Blend products $p \in \mathbf{P}$
- Properties $q \in \mathbf{Q}$
- Time points $t \in \mathbf{T}$
- Units $j \in \mathbf{J}$
- Stream properties $\sigma_{s,q}$
- Product property specifications $\sigma^U_{p,q}/\sigma^L_{p,q}$
- Stream availability/product demands



Determine

- Material flows $F_{j,j',t}$
- Material transfer binary $X_{j,j',t}$

Objective

- Minimizing operating cost
- Maximizing profit

Binary variable $X_{j,j',t}$

- No simultaneous feeding/withdrawing from pools

$$X_{j,j',t} + X_{j',j,t} \le 1 \quad \forall j \in \mathbf{J}^{\mathrm{B}}, j', t$$

$$F_{j,j',t} \le M \cdot X_{j,j',t} \quad \forall j, j', t$$

- Fixed cost for pumping

Model Formulations

Concentration model

Variables

$F_{j,j',t}$: Flow

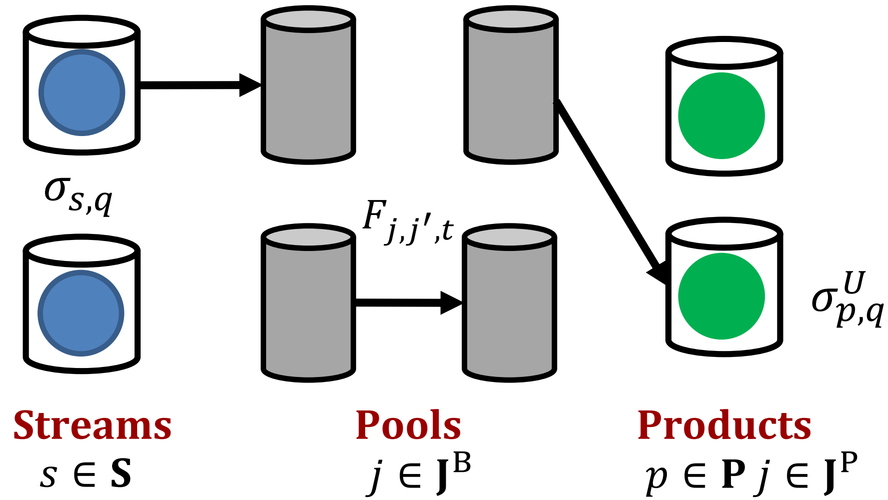
$I_{j,t}$: Inventory

$C_{q,j,t}$: "Concentration" of property

Key constrains

$$C_{q,j,t-1} \le \sigma^U_{p,q} + M\left(1 - X_{j,j',t}\right) \ \forall j,p,j' \in \mathbf{J}^{\mathrm{P}}, q, t$$

$$I_{j,t} \cdot C_{q,j,t} = I_{j,t-1} \cdot C_{q,j,t-1} + \sum_{in}(\dots) - \sum_{out}(\dots) \quad \forall j \in \mathbf{J}^{\mathrm{B}}, q, t$$



Split fraction model

Variables

$\bar{F}_{q,j,j',t}$: Property flow

$\bar{I}_{q,j,t}$: Property inventory

$\xi_{j,j',t}$: Fraction for material transfer

Key constrains

$$\bar{F}_{q,j,j',t} \le F_{j,j',t} \cdot \sigma^U_{p,q} \quad \forall j,p,j' \in \mathbf{J}^{\mathrm{P}}, q, t$$

$$\bar{I}_{q,j,t} = \bar{I}_{q,j,t-1} + \sum_{j'} \bar{F}_{q,j',j,t} - \sum_{j'} \bar{F}_{q,j,j',t} \ \forall q,j,t$$

$$\bar{F}_{q,j,j',t} = \xi_{j,j',t} \cdot \bar{I}_{q,j,t} \ \ \forall q,j,j',t$$

$$F_{j,j',t} = \xi_{j,j',t} \cdot I_{j,t} \ \forall j,j',t$$

Source based model

Variables

$\tilde{F}_{s,j,j',t}$: Source flow

$\tilde{I}_{s,j,t}$: Source inventory

Key constrains

$$\sum_{s} \tilde{F}_{s,j,j',t} \cdot \sigma_{s,q} \le F_{j,j',t} \cdot \sigma^U_{p,q} \quad \forall j,p,j' \in \mathbf{J}^{\mathrm{P}}, q, t$$

$$\tilde{I}_{s,j,t} = \tilde{I}_{s,j,t-1} + \sum_{j'} \tilde{F}_{s,j,j',t} - \sum_{j'} \tilde{F}_{s,j,j',t} \ \forall s,j,t$$

$$I_{j,t} = \sum_{s} \tilde{I}_{s,j,t} \ \ \forall j,t$$

$$\tilde{F}_{s,j,j',t} = \xi_{j,j',t} \cdot \tilde{I}_{s,j,t} \ \forall s,j,j',t$$

Solution Strategies

Remarks

- All three formulations mentioned are nonconvex MINLP, due to the bilinear terms in equality constraints and binary variables
- There are also "hybrid" formulations where people applied *some* linear constraints in one formulation to another formulation
- Instead of directly sending this MINLP to the general purpose MINLP solver, people aim to propose specific solution strategies

MINLP; Bilinear terms w/ two continuous variables; Minimizing;

Approach 1

Step 1. Discretize one variable in bilinear terms; Reformulate an MILP with reduced feasible space (=>UB)

Step 2. Relax the MILP in Step. 1 through some relaxation techniques, or linearize the original MINLP (=>LB)

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More details: My previous system seminar talk; Kolodzeij et al. (2013a, 2013b);

Notes:

1. Convex hull, facet-defining inequalities associated with the MILP in Step. 1 are comparably well studied, see Gupte et al. (2013)
2. The size of the MILP in Step 1 limits its application to small instance (Kelly et al., 2017)

Solution Strategies

MINLP; Bilinear terms w/ two continuous variables; Minimizing;

Approach 2

Step 1. Solve a relaxed problem (MILP) to the original MINLP (=>LB)

Step 2. Start from the solution obtained in Step 1, try to *construct* a feasible solution to the original MINLP (=>UB)

……

Note: General purpose MINLP solver also perform step 1 using convex relaxation. However, often time people simply drop *all* nonlinear constraints

Basic idea

Decompose the blend scheduling problem into two sub problems: Logistics and properties

Logistics: Material balance, unit capacity, operating rules….

Properties: Product specifications

Phenomenological decomposition heuristic (PDH), see Menezes et al. (2015)

Pros: A "good" feasible solution can be obtained in a short time

Implementation Example

Here, we briefly introduce an implementation from Lotero et al. (2016)

Step 1. Drop all nonlinear constraints, solve the resulting MILP

Step 2. Fix all binary variables in the original MINLP to the value obtained from Step 1, solve the resulting NLP to local optimal

Step 3. Optimality check. When necessary, add *cuts* to exclude the integer combinations used in Step 2, then go to Step 1

……

Some reported computational performance:

Lotero et al. (2016): "… …only needs one iteration and less than 5 s to find a good feasible solution. Neither BARON 14.0, SCIP 3.1 or ANTIGONE 1.1 are able to find a solution in 30 min when solving the original MINLP formulation"

Kelly et al. (2017): MILP in Step 1 has ~8000 continuous variables, ~3000 binary variables, ~20,000 constraints, and is solved in 200 s; NLP in Step 3 has ~20,000 continuous variables, ~20,000 constraints, and is solved in 20s. Gap between two solutions is within 0.09%

Solution refinement after Step 1: See Li et al. (2012)

Another decomposition strategy: Inventory pinch, see Castillo et al. (2016)

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